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HEAT PROCESSING OF BEEF. IV. FUNCTIONAL RELATIONSHIPS OF TEMPERATURE, TIME, AND SPACE DURING PROCESSING AT HIGH RETORT TEMPERATURES[a]

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1. THE APPLICATION OF MATHEMATICS TO PRESERVING PROCESSES—A RECAPITULATION OF PRINCIPLES, OBJECTIVES, AND NOMENCLATURE

The necessity of application of the fundamental laws and practices of mathematics, bacteriology, engineering, physics, statistics and chemistry is becoming more and more evident to research workers investigating and developing preserving processes. Whereas in the past it was deemed sufficient to follow a few rules established by tradition and experience, at the present time a more fundamental approach is required to assure a product of highest possible quality and acceptability and at the same time one which is safe for consumption.

The place for application of the fundamental principles seems to be in the investigation of both the raw materials processed and of processing methods. It is difficult to differentiate between these two categories as methods depend on materials processed. It is possible, however, to group the raw materials in definite units according to their properties and develop general methods applicable to more than one commodity. Therein lies the paramount importance of fundamental research, the importance of investigation of fundamental laws which govern not one process but a whole domain of various phenomena which can be explained by the same basic principle applied to varying conditions. Once the fundamental laws are established, the prediction of the course of a process is facilitated without recourse to repeated experimentation, although additional data pertaining to the properties of materials processed and process conditions are required for prediction.

Mathematics furnishes means for expression of the fundamental laws and principles in a relatively short and precise manner whenever relationships of variable factors are involved. Moreover, seemingly unrelated physical phenomena can be expressed in the same general mathematical form.

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Mathematical development of basic laws leads to the solution of specific problems for a specific set of conditions. The mathematical solution then can be verified by actual experimentation.

When pure (theoretical) mathematics fails or where theory cannot be applied readily to the problem in question, empirical relationships must be established from experimental data. The thinking process is reversed and instead of experimental verification of a postulated theoretical relationship, an empirical relationship is established from experimental data at hand. The empirical prediction equation may be derived in more than one way; a theoretical equation may be modified by including a correction factor or an equation of a different mathematical form may be deduced. Methods of establishing the equations differ widely with the field of research.

Statistical methods offer excellent tools in deriving prediction equations by means of least squares regression methods which give the best fitting curves once the form of functional relationship is postulated. Another advantage of statistical methods lies in the determination of experimental errors and sources of variation; these methods are of especial value in biological sciences where large variations are encountered in experimental materials. The use of statistical methods results in much better knowledge of precision in prediction as compared with visual curve fitting methods widely accepted in many fields of investigation. Where precision of determination is of primary importance the use of statistical methods is a practical requirement. Statistical design of an experiment facilitates greatly the interpretation of results and in most cases, saves labor and cost. However, in some cases a visual estimate is sufficient for practical purposes. It is up to the research worker to decide which tools to use. The degree of precision of experimental measurements should be a deciding factor. Unprecise measurements do not warrant derivation of equations representing the smallest detail of the experimental data as such prediction will not be precise. In engineering practice it is quite common to use the visual method of curve fitting; the relationships thus established have been found to be satisfactory for the practical purposes for which they were used. Visual curve fitting is also in common use in the field of food technology. It seems, however, that more rigorous methods of deriving of functions are generally of more value.

Experimental errors. Any investigation presents the problem of validity and precision of the results and should include consideration of the experimental errors. Experimental errors may be determined for the means of a number of observations and will be found to decrease as the size of the sample increases, thus the precision of the experimental results will usually be higher if more determinations are made. Another question arises in determination of experimental errors of results not directly measured but derived by mathematical formulae and depending on measurements of other variables; such errors are found to be dependent on errors in measurement of the component variables and from theoretical considerations one is able to estimate the precision of the result as a function of the partial errors in determination of these variables *(5)*. Standard statistical methods are used to estimate experimental errors connected with direct observations of various measurable properties. The estimate of the experimental error enables one to establish at defined probability level limits within which one expects the results to be contained. These limits derived by rigorous methods indicate both the precision and reliability of the results.

In the past the consideration of experimental errors has been often disregarded in many fields of research; in other fields the considerations of experimental errors was not treated rigorously, often because there was no rigorous method available for such a treatment or because it was not considered important. However, any measurements even of homogeneous and "ideal" materials do involve variations from one observation to another, and when biological materials are tested an inherent variation within the material itself is added to the variations in measurement data. Therefore, results which do not contain estimates of experimental errors and their sources cannot be considered reliable and valid. An estimate of experimental errors and their sources should be included in the results of an investigation to give an idea of the precision of the experiment. In some cases, however, it might not be possible to estimate the errors by rigorous methods and only an approximate evaluation of errors is possible of attainment. The errors can be estimated from theoretical considerations of the propagation of precision indices *(5, 17)* when functional relationships are used to calculate results and standard statistical methods can be used if available for estimation of directly measurable properties.

Heat processing of meat. In the study of heat processing of meats as in all food preservation processes two objectives are to be attained: a palatable product acceptable to the consumer and a product which will not endanger the health or life of the consumer. These objectives have to be achieved simultaneously since attainment of one of them without the other renders the product unmarketable. To reach the above goals two general problems have to be considered: (a) the properties of the raw and processed material (physical, chemical and bacteriological); and (b) methodology of processing (specification of process in terms of length and processing temperature, and sterilizing value).

Three types of approach to such problems are conceivable:

(1) Purely practical approach based on past experience and tradition without any consideration for fundamental principles and laws behind the phenomena investigated.

This type of approach has been used in the past with varying degrees of success for various commodities. It has been quite successful in some cases, but often it has proved to be useless as far as achieving both of the above objectives. The development of the food industry and the increasing complexity of problems facing it has resulted in recognition of the importance of fundamental laws governing the heat processing of foodstuffs.

(2) Purely theoretical investigations based on idealizing assumptions and ideal materials. Very little consideration is given to the practical situation and highly hypothetical conditions are studied with little regard for the possible application to the solution of practical problems.

Upon cursory examination, this type of approach does not seem to be justified as it does not furnish answers to specific practical problems. In the long run, however, the understanding of fundamentals may prove absolutely necessary to provide a solution to complex problems. Over-emphasis on the theory may prove just as dangerous as disregard of it.

(3) A combination of the first two approaches involving an investigation of the fundamental laws governing the process with a view to the application of these fundamentals to practical conditions. An investigation is planned from the beginning to provide an answer to practical problems by means of ascertaining the basic fundamental principles involved, neither aspect being neglected. The precision of the results is taken into consideration by estimating the experimental errors which occur during the experimental phase of the research.

The investigation of canning processes represents a good example of application of fundamental principles to an actual manufacturing problem; it also serves as a good example of contributions from various branches of science to a problem in the field of food technology.

Review of previous investigations. A considerable amount of work on the fundamentals of heat processing has been done in the past. Little concern, however, has been shown for the evaluation of the precision of the results. A very meager amount of data available pertains specifically to meat and hardly any to beef. Under these circumstances, the applicability of general laws to processing of beef might be doubted. Actual experimentation and verification of the fundamental principles with the experimental data by means of rigorous estimate of the precision of results may decide whether the doubts have been justified. Experimental data are lacking for high-temperature process, as well as verification of cooling phase theory.

The small amount of information available in the literature concerning physical characteristics of meat and especially beef has been reviewed by Hurwicz *(9)* and by Bard *(3)*. Bard and Tischer *(4)* furnished additional data on objective evaluation of quality of beef connected with such physical characteristics as tenderness, moisture content, and fiber cohesiveness. Hurwicz and Tischer *(11)* discussed some theoretical considerations in heat processing of beef in cans in detail. The same authors *(12)* studied the temperature distributions in beef sealed in cans during thermal processing at two processing temperatures and determined the thermal diffusivity under these conditions. They discussed in detail experimental errors associated with the prediction equation derived and in various determinations made in the course of the experiment.

The situation is quite different in the field of evaluation of thermal processes necessary to assure commercial sterility of various commodities. A wealth of information is available in the literature pertaining to process determination and several methods have been worked out to ascertain the sterilizing value of thermal processes. Bard *(3)* and Hurwicz *(9)* reviewed some of these methods and investigations.

In the last decade a new approach to the problem of determination of an adequate process has been developed by Stumbo *(14, 15)*, Hicks *(7, 8)*, and Gillespy *(6)* and investigated by Hurwicz and Tischer *(12)*. The methods based on the new concepts involve a change in emphasis from the center location in the can to the whole can volume as a criterion of the adequacy of a process and the use of thermal death rate data for ascertaining the degree of sterility of cans rather than thermal death time data which suggests a total destruction of bacterial populations. To apply the newer concepts to the actual heat processing of various commodities it is necessary to have detailed data on temperature distributions within cans packed with these commodities, and to derive the sterilizing value distributions from the temperature distributions. Such data are not available for actual canned foodstuffs except for the work of Hurwicz and Tischer *(12)*. Thus far in the case of some commodities the "safe" process resulted in an acceptable product, but in other cases and notably of beef, the "safe process" did not result in a canned product of high acceptability.

Objective evaluation of quality. Whereas the fundamental laws of heat transfer and thermobacteriology were applied to the problem of process evaluation, no fundamental laws can be found applying to the problem of objective evaluation of the quality of meat in terms of its tenderness and juiciness. No functional relationships were postulated relating the conditions of thermal process (processing temperature, time of processing) with the resulting shear force of the product or the drained juice and the lethality of the process. Very few references are available correlating even any two of the above factors *(4)*. A need for establishing such functional relationships as an introduction to establishment of general fundamental laws seemed to be urgent. Part III *(16)* of this series described an investigation intended to fulfill such needs.

A continuous failure to arrive simultaneously at a commercially sterile, palatable product when processes at relatively low temperatures (212-250° F) are employed leads one to believe that perhaps a high-temperature short-time process would ultimately produce a better product. No information was found available in the literature pertaining to the high-temperature process applied to meat products.

Very few investigations *(7, 12, 13)* concerned themselves with evaluation of experimental errors encountered in food research.

It seems then that a serious need exists to enrich and develop the present data on thermal and other physical characteristics of meat, to develop further the application of fundamental principles of heat conduction to the heat processing of beef during all phases of the thermal process, to correlate the physical changes occurring during heat processing with the lethality requirements and thus to obtain a new method of process determination for a safe and palatable product, and finally to compare the methods of visual curve fitting and those of regression techniques in the light of experimental errors. Exploration of a wide range of processing temperatures seems to be of especial value. An investigation of the bacterial distributions within the container would help greatly in testing the volume lethality concepts; this last investigation would, however, be beyond the scope of this study.

Objectives of the Investigation

In the light of the preceding discussion, the objectives of this investigation were set up as follows:

1. To determine some thermal properties of beef (thermal diffusivity, characteristics of heating and cooling curves) and temperature distribution during processing.
2. To conduct the investigation at a wide range of temperatures to include and to emphasize the high retort temperatures close to 300° F.

3. To investigate heating and cooling process with the emphasis on the phase starting after the steam is shut off and cooling water started.
4. To obtain empirical relationships among variables encountered in heat processing and to determine and correlate the physical changes in beef with process requirements for a sterile and acceptable product.
5. To compare the visual method of curve fitting with the statistical regression method of determination of slopes of heating and cooling curves.

The general purpose was to establish wherever possible, the fundamental laws expressed in mathematical form, or to arrive at mathematical empirical functional relationships embracing as large a realm of conditions as possible. This is to be achieved by statistical methods of evaluation and with emphasis on experimental errors associated with derived expressions.

The authors in the published three parts of this series attempted to follow the above established objectives. Part I *(11)* serves as an introductory paper explaining in detail the fundamental laws of heat conduction and deriving step-by-step the heat conduction equation commonly used in food technology applications. In Part II *(12)* the authors investigated thermal properties of beef canned in No. 2 cans, arriving at empirical time-temperature-space relationships during the heating phase of processing by the use of statistical methods. The experimental errors were determined and discussed. The relationship between the theoretical predictions resulting from the fundamental laws and the experimental results was ascertained. In Part III *(16)* the investigation was extended to higher than conventional temperatures up to 315° F., and the results of the objective evaluation of the quality were reported by means of mathematical correlation of the quality attributes with other process variables (time and temperatures). Again, attempts were made whenever possible on establishment of fundamental laws governing such functional relationships.

Part IV of the series, presented in this paper, is an investigation of the fundamental laws governing the thermal process of beef extended to high temperatures in 300 x 308 cans. Empirical functional time-temperature-space relationships are obtained for both cooling and heating phase for an extremely large number of determinations in the whole space of the container. The equations resulting from this investigation are obtained by rigorous statistical methods and carry an experimental error estimate. In the authors' opinion, the information contained therein as in the papers to follow, contributes to the knowledge of beef properties specifically as well as to the knowledge of high temperature processes. The information is obtained by rigorous statistical methods and thus is believed to be reliable, eliminating personal bias from the evaluation. The next paper in this series will cover the temperature distributions in the processed cans with an analysis of their occurrence. (Part V). Thermal diffusivity and "slopes" of heating and cooling curves (f_h and f_{c1}) and their determinations will be discussed in the following paper of the series. The intermediate cooling phase starting immediately after the steam shut-off will be the topic of another paper; and finally the sterilizing effect investigation will be reported in the last paper of the series. All these investigations were carried out for high-temperature processes.

NOMENCLATURE

a — radius of the container — (cm).

$a_{11} = A_{11}J_0(\mu_1 r) \sin \lambda_1(z+l)$ — intercept coefficient of the heating curve Eq. 2c, in first approximation of the theoretical equation, in semi-log plot; or the antilog of the intercept of log $[(RT-CT)/(RT-IT)]$ axis plotted vs t when $t=0$ — (dimensionless).

$\hat{a}_{11}$ — intercept coefficient determined from regression calculations of the heating curves from experimental data.

$\hat{a}'_{11}$ — intercept coefficient determined from Eq. 6a using calculated $\hat{j}$ values from cooling curves.

$b' = +k(\mu^2_1+\lambda^2_1)$ — slope of the heating curve on semi-log — (min.$^{-1}$).

$b_h = b' \log e$ — same as above for logarithm to base 10.

$\hat{b}_h$ — same as above determined by regression calculations from experimental data.

b'' — slope of the cooling curve as defined in Eq. 5a for semi-log plot (min.$^{-1}$).

$b_{cl} = b'' \log e$ — same as above for logarithm to base 10.

$\hat{b}_{cl}$ — same as above determined by regression calculations from experimental data.

f_h — "slope" of the heating curve as commonly used, actually reciprocal of $-b_h$ — (min.).

f_{cl} — same as above for cooling curves.

$g = (RT-CT)_{t_1}$ — temperature difference between the retort and can at the end of the heating and beginning of cooling — (°F.).

j — intercept coefficient of the cooling curve Eq. 5c, or the antilog of the intercept of log $[(CT-TC)/(CT_{t_1}-TC)]$ axis plotted vs $(t-t_1)$ when $(t-t_1)=0$ for all locations in the can. Also defined in Eqs. 5a and 6a (dimensionless).

$\hat{j}$ — intercept coefficient of the cooling curve determined by regression calculations from experimental data.

$\hat{j}_1$ — intercept coefficient of the cooling curve determined from Eq. 6a using $\hat{a}_{11}$ values.

k — thermal diffusivity — (cm^2 × min.$^{-1}$).

$2l$ — height of the can — (cm).

$m = (CT-TC)$ — temperature difference between the can and the cooling water — (°F.).

r — cylindrical coordinate (distance from central vertical cross-section of the can) — (cm).

t — time — (min.).

t_1 — time when heating ends and cooling begins — (min.).

z — cylindrical coordinate (distance from central horizontal cross-section of the can) — (cm).

$x_{cl} = (t-t_1)$ — (min.).

$A_{im} = -4[(-1)^m-1]/m\pi(\mu_i a)J_1(\mu_i a) = 4(1-\cos m\pi)/m\pi(\mu_i a)J_1(\mu_i a)$ — constant — dimensionless).

$C_1 = \hat{a}_{11}/a_{11}$ — heating lag coefficient (based on heating data) — (dimensionless).

$C_2 = \hat{a}'_{11}/\hat{a}_{11}$ — cooling lag coefficient (based on heating data) — (dimensionless).

$C_3 = \hat{j}/j$ — cooling lag coefficient — (dimensionless)

C.V. — coefficient of variation — (%).

CT — can temperature at a location — (°F.).

IT — initial temperature throughout the can — (°F.).

RT — retort (or processing) temperature — (°F.).

TC — cooling bath temperature — (°F.).

$J_0(\mu_i r)$ — Bessel function of first kind, zero order.

$J_1(\mu_i a)$ — Bessel function of first kind, first order.

$(\mu_i a)$ — positive zeros infinite in number $(i=1, \ldots, \infty)$ of $J_0(\mu_i a)=0$.

μ_i — positive roots infinite in number $(i=1, \ldots, \infty)$ of $J_0(\mu_i a)=0$.

$\lambda_m = \dfrac{m\pi}{2l}$ — constant depending on can size — (dimensionless).

Subscripts and other symbols

i — numbers $1, \ldots, \infty$ denoting the number of terms in series Eq. 1.

m — numbers $1, \ldots, \infty$ denoting the number of terms in series Eq. 1.

h — denotes values pertaining to the heating phase, or determined during that phase.

cl — denotes values pertaining to the cooling phase, or determined during that phase.

t_1 — denotes values of variables at time $-t_1$.

^ — caret — used to denote experimental values determined by regression calculations.

2. FUNCTIONAL RELATIONSHIPS OF TEMPERATURE, TIME, AND SPACE DURING PROCESSING AT HIGH RETORT TEMPERATURES

The thermal characteristics of canned product such as thermal diffusivity of material, patterns of heat penetration and temperature distributions during heat processing with all their secondary effects exert considerable influence on the ultimate quality of the product. These characteristics depend on several variables the importance of which was investigated during the course of this experiment. The assumptions made during the investigation were discussed in Part I and II of this series *(11, 12)* and are the same as in *(11)*. The theoretical relationship referred to in several instances below is reproduced here as follows:

$$\mathrm{CT} = \mathrm{RT} - (\mathrm{RT} - \mathrm{IT}) \sum_{i=1}^{\infty} \sum_{m=1}^{\infty} A_{im} J_0(\mu_i r) \sin \lambda_m (z + l)\, e^{-k(\mu_i^2 + \lambda_m^2)t} + \qquad \text{Eq. 1}$$

$$\begin{cases} 0 & \text{for } 0 < t < t_1 \\ (\mathrm{TC} - \mathrm{RT}) \left[1 - \sum_{i=1}^{\infty} \sum_{m=1}^{\infty} A_{im} J_0(\mu_i r) \sin \lambda_m (z + l) e^{-k(\mu_i^2 + \lambda_m^2)(t - t_1)} \right] & \text{for } t > t_1 \end{cases}$$

where CT is the can temperature at a location; RT is the retort temperature (or processing temperature); IT is the initial temperature throughout the can; and TC is the cooling bath temperature; and temperatures can be measured either in °F. or °C. as long as consistent. A_{im} is a constant depending on the initial temperature distribution in the can assumed to be uniform.

$$A_{im} = \frac{-4\,[(-1)^m - 1]}{m\pi\,(\mu_i a)\,J_1(\mu_i a)} = \frac{4\,(1 - \cos m\pi)}{m\pi\,(\mu_i a)\,J_1(\mu_i a)}$$

$J_0(\mu_i r)$ is the Bessel function of first kind, zero order which can be found in tables of Bessel cylindrical functions, and which is equal to 1 in the center at $r = 0$.

$J_1(\mu_i a)$ is the Bessel function of first kind, first order and

$(\mu_i a)$ = positive zeros infinite in number of $J_0(\mu_i a) =$ or

μ_i = positive roots infinite in number of $J_0(\mu_i a) =$

all of which can be found in tables of Bessel functions, and t is time; t_1 is time when heating is terminated and cooling starts; r, z are cylindrical coordinates of space with the origin in the center of the can; a is radius of the can; $2l$ is total height of the can; k is thermal diffusivity; and

$$\lambda_m \text{ is } \frac{m\pi}{2l}$$

Note that $\sin \lambda_m (z + l) = 1$ in the center of the can, which results in $A_{im} = j$ (as defined by Ball *(2)* for heating curves) at the center of the container where $J_0(\mu_i r) = 1$.

The values of coefficients and constants for the first term of the expansion of Equation 1 (for a 300 x 308 can) are given in Table 1 in which $A_{11} J_0(\mu_i r) \sin \lambda_1 (z + l) = a_{11}$ was defined as the intercept coefficient for a given location.

It has been found in the past by numerous research workers that the use of the first term of the converging series in Equation 1 resulted in good approximation of experimental data with the exception of the first few

TABLE 1

Distribution of theoretical intercept coefficients (a_{11}) of heat conduction equation, and values of can constants for 300 x 308 can

$+z$[1] (cm)	r (cm)[2]			
	0	1	2	3
0.00	2.041	1.986	1.825	1.572
1.00	1.900	1.849	1.699	1.404
2.00	1.495	1.455	1.337	1.152
2.75	1.054	1.026	0.9426	....

$\mu_1 = 0.3294$ $\lambda_1 = 0.3740$ $(\mu^2_1 + \lambda^2_1) = 0.2484$

[1] Distance from the central horizontal cross-section.
[2] Distance from the central vertical cross-section.

minutes of the heating and cooling phase. The deviation from experimental results during the induction period of the heating phase did not influence the sterilizing value of the process, and may be considered negligible. The reverse is true when deviations (due to the use of only the first term of the expansion) from the actual cooling curve are considered which occur in the early stages of cooling. This period will be called henceforth the "intermediate" cooling period. The later stages of the cooling were found to be well approximated by the use of the first term of the theoretical expansion. These findings were restricted to determinations at the geometric center of the container.

The investigation of the whole space of the container indicated *(12)* the same trend for the heating phase. The experimental results of this investigation extended these findings tentatively to the whole thermal process when 1512 heating curves and 1512 cooling curves were plotted on semi-log graph paper for 25 locations in the can and for six processing temperatures. With the exception of the "intermediate" cooling phase the plot resulted in straight line relationships of temperature and time for all of the locations. Such relationships are expected when only one term of the theoretical equation is retained. In view of the above, regression equations were derived for the heating and cooling (with the exception of "intermediate" cooling) curves on the assumption of linearity of the logarithm of temperature differences with respect to the time.

The objective of the present work was to determine empirically the functional relationships existing among temperature, time, and space during processing of beef at high retort temperatures.

EXPERIMENTAL PROCEDURE

Apparatus and equipment. The containers used in this experiment were 300 x 308 steel, tin-plated cans sealed under 25 in. vacuum and were processed in a custom made retort of vertical type (11½ in. inside diameter and 17½ in. height).

Temperatures were determined by means of copper-constantan thermocouples and recorded by Brown Electronic Potentiometer described and calibrated by the authors in the past *(7)*.

Design of the experiment. *Raw product.* One of the ultimate practical objectives in an investigation of canning processes is to increase the palatability of a product. The choice of raw product is of importance in achieving this objective. The raw product chosen in this investigation was round of beef (matched sets of inside, outside, and knuckle of round) canner and cutter grade. This cut is likely to yield a palatable product and at the same time it is economically acceptable to the canner. Thirty-six matched sets of round were used during the course of the experiment, supplied weekly via Railway Express from the Armour Company plant in Chicago. The meat was shipped in special insulated containers refrigerated from within by means of cans with ice and was stored at 32° F. for periods of time not exceeding 4 days.

From each cut (inside, outside, and knuckle) 9 cylindrical pieces of approximately the size of the can used (each weighing 300 g.) were cut and used as follows:

(a) One piece of meat was sealed raw and quick-frozen at −50° F. for bacteriological determinations *(1)*;

(b) two pieces were used to measure simultaneously temperatures at 3 locations in the can and to determine shear force and amount of drained juice *(8)*;

(c) two pieces were used to measure temperatures at 4 additional locations in the can;

(d) two pieces were used for bacteriological determinations *(1)* (quick-frozen after processing); and

(e) two pieces were used for storage at 100° F.

Half of the above sample (with the exception of the raw frozen can) was processed at one of the designated temperatures to give thermal histories at 7 different locations in the can, whereas the other half was processed at another temperature. Thus every week 3 cuts were used and 6 processing runs were made at all 6 designated processing temperatures.

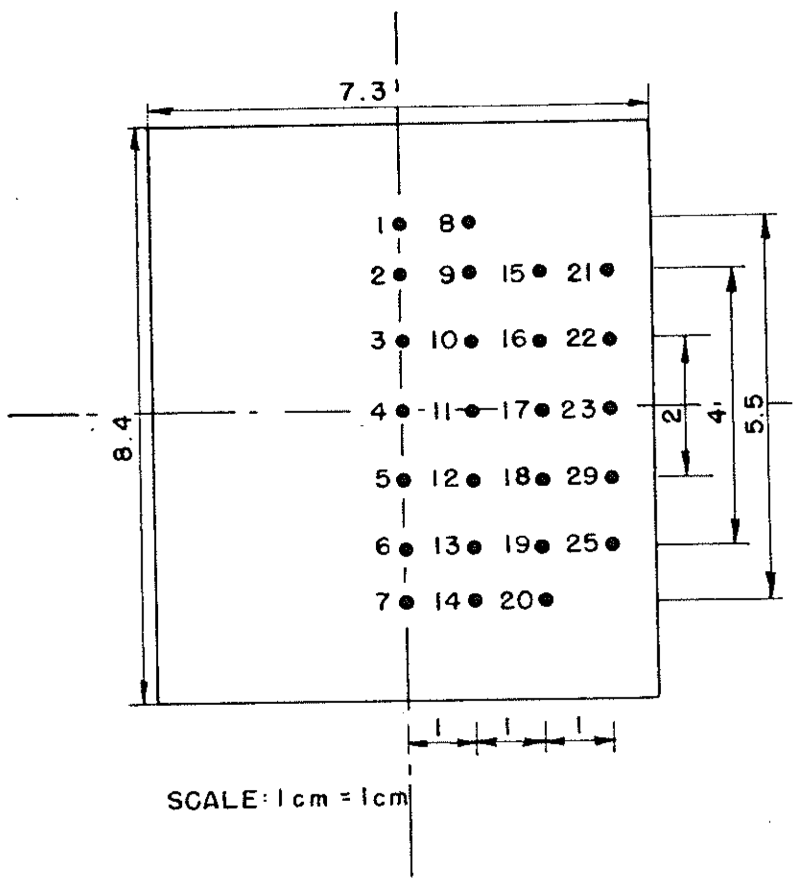

Figure 1. Distribution of locations tested during the investigation. (Vertical cross-section through the central axis of 300 x 308 cans.)

The variability of biological materials has been emphasized by many writers and in the specific case of beef a detailed discussion of real and ideal conditions was made by Hurwicz *(3)*.

To avoid variations caused by unhomogeneity of meat and by other factors affecting the raw product (such as differences in feed and age, type of cut and grade, or storage conditions) the investigation was limited to only 3 cuts of round of beef. One solid chunk of meat was processed per can (to reduce influence of convection) with its fibers parallel to the central vertical axis of the container. The storage period was held uniform throughout the experiment. Furthermore, all temperature treatments were used on one round from one animal.

Statistical design. Effects of processing temperatures, processing times, and combinations thereof on thermal, thermobacteriological and other physical characteristics of processed beef were to be determined in the course of this investigation. The wide range of processing temperatures (RT) from 225-315° F. chosen, enclosed for practical purposes, the possible range to be used in canning and offered a comparison between low- and high-temperature processing. The processing times ranged from 2-24 minutes at 315° F. to 20-120 minutes at 225° F. The time-temperature combinations used were shown in Figure 1 of Part III of this series *(8)*. The times corresponding to the time set values — T were calculated on the basis of previous research *(4)* on beef (shoulder clod) sealed in No. 2 cans which furnished some data on F_o value, and so proportionally lower processing times were assigned to higher processing temperatures to yield the same F_o value within one time set. Since the data available covered only a small portion of the range considered for this experiment, extrapolation to higher temperatures was necessary. Figure 1 *(16)* was established partially from the above considerations. Since the raw material and containers used by Bard and Tischer *(4)* differed from those used in the present investigation, and since extrapolations had to be made to obtain estimates of F_o values at higher temperatures, it was decided to use Bard and Tischer's data only as a rough guide for time set value determination. Two-minute intervals at 315° F. retort temperature from 2 to 24 minutes, and ten-minute intervals at 225° F. retort temperature from 10 to 120 minutes, were taken in actuality at the extremes of the temperature range to serve as end points for the time set values from 1 to 12. The times corresponding to other temperatures were interpolated from the lines connecting the end points for each of the time set values. The details of the design were shown in Tables 1 and 2 of Part III of this series *(16)*.

The information on the effects of individual cuts had to be sacrificed to obtain relatively more information on the effects of temperature and time on the properties of the whole round, because of the limited number of samples which could be taken from one round of beef (one animal) or from one of the cuts composing a round, and because of a wide range of temperatures and times to be used. Therefore, for every time set value the effects of all temperatures were tested for the 3 cuts together, but not all temperature-time effects were tested within every individual cut. This disadvantage of the design was compensated by the fact that it could be used on the row-by-row basis and could be interrupted at any row without complicating the interpretation of the results and detracting from the validity of the conclusions. The possibility of interruption at any time had to be considered since no information was available as to how far one should go in increasing the processing times to obtain useful information.

The statistical model of the design which may be interpreted as an incomplete block design is as follows:

$$y_{ij_{(k)}} = \mu_{(k)} + s_{ij}\left[b_{i_{(k)}} + T_{j_{(k)}}\right] + e_{ij(k)}$$

where $s_{ij} = 1$ if T_j occurs in block i

and $s_{ij} = 0$ if T_j does not occur in block i, or

$$\begin{bmatrix}\text{attribute value}\\ \text{for the row}\end{bmatrix} = \begin{bmatrix}\text{mean of}\\ \text{the row}\end{bmatrix} + s_{ij}\begin{bmatrix}\text{block effect}\\ \text{in the row}\end{bmatrix.+ \begin{matrix}\text{treatment effect (time-temp.}\\ \text{combination in the row)}\end{matrix}\Bigg] + \begin{matrix}\text{exp. error}\\ \text{in the row}\end{matrix}$$

and the analysis can be made row by row (representing the time set value). In this case the retort time-temperature interactions do not have to be evaluated separately as each time-temperature combination (a treatment T) is considered individually. The advantage of such an analysis lies in the fact that any attribute such as shear force, drained juice or sterilization value may be evaluated in terms of its deviation from the postulated equivalency in terms of a time-set value or attribute A = f (T) = f (t, T). Once a functional relationship is established for the attribute A (e.g., shear force) in terms of the variables involved (of whatever mathematical form it might be) and the time set values determined to result in equivalent values of this attribute for different combinations of variables, the observed deviation from equivalency can be tested by standard statistical methods in a row-by-row analysis.

Heat processing. *Technique.* The experimental routine techniques followed during the investigation of thermal characteristics of beef during heat processing were basically the same as followed by Hurwicz in previous research *(4)*, with some modification.

The cut of meat was trimmed of excessive fat, skin, and connective tissue; the 300 x 308 cans were filled as described above. The assembly of can and thermocouples was made. Net weight of meat was recorded, and then the weight of meat and assembled can was also determined before and after sealing. The thermocouples in the can were connected to the potentiometer wires, and the retort closed. The time pattern temperature and pressure controls including the automatic timer were put into operation, and all vents were closed (the drain was left partially open to allow for removal of the condensate). Venting during the processing was through the vent valve activated automatically by the controls. The length of the come-up and blow-down time (cooling under pressure) was regulated by the time patterns designed for the process, and the processing time by the automatic timer setting. The controls were shut off, and processing terminated when the temperature of the point of greatest heating lag was below 100° F. The retort was opened and drained, and the cans weighed to check for possible losses or gains during the processing period. The curves from potentiometer charts were transcribed and then replotted in rectangular and semi-logarithmic coordinates. Thermal diffusivities were determined from the heating curves.

TABLE 2

Intercept coefficients ($\hat{a}_{11}$) for six retort temperatures determined by regression method

Location	Retort Temperature (°F.)						
	225	243	261	279	297	315	Average
1	0.732	0.907	0.829	1.560	1.575	4.091	1.619
7	0.805	0.759	1.209	0.974	1.005	1.256	1.001
8	1.008	1.235	1.320	1.564	1.101	1.552	1.297
14	0.373	1.409	1.572	1.095	1.300	1.232	1.164
20	0.735	1.145	1.670	1.523	1.106	1.594	1.296
21	0.767	0.821	0.814	0.816	1.110	1.352	0.947
22	0.454	0.997	1.056	1.130	1.229	0.917	0.964
23	0.810	1.032	1.087	0.964	1.167	0.855	0.986
24	0.611	0.903	1.165	1.059	1.322	0.853	0.986
25	0.697	0.643	0.554	0.937	1.351	0.663	0.808
2	1.069	1.334	1.622	1.348	2.241	1.541	1.526
6	0.902	1.533	1.673	1.473	1.310	1.324	1.369
9	1.075	1.401	1.886	1.613	1.708	2.141	1.637
13	1.618	1.890	1.923	1.355	2.390	1.600	1.798
15	0.775	1.442	1.605	1.294	1.112	1.209	1.240
16	1.348	2.082	1.653	1.601	1.241	1.211	1.524
17	1.267	1.363	2.230	2.109	1.521	1.261	1.625
18	1.438	1.592	1.859	1.434	1.336	1.260	1.487
19	1.288	2.123	1.456	1.449	1.231	1.304	1.475
3	1.380	1.355	1.729	1.620	1.465	1.083	1.439
4	1.463	1.3448	1.6295	1.4535	1.4453	1.2860	1.437
5	1.959	2.272	2.339	2.022	1.414	1.793	1.967
10	1.240	1.307	1.255	2.813	1.930	1.669	1.707
11	1.109	1.353	1.724	1.374	1.321	1.248	1.355
12	1.496	1.615	2.514	1.413	2.081	1.527	1.774

TABLE 3

Distribution of intercept coefficients ($\hat{a}_{ij}$) in 300 x 308 can

r (cm)	z (cm)						
	2.75	2.00	1.00	0.00	−1.00	−2.00	−2.75
				RT = 225° F.			
0	0.732	1.069	1.380	1.464	1.959	0.902	0.805
1	1.008	1.075	1.249	1.109	1.496	1.618	0.373
2	....	0.775	1.348	1.267	1.438	1.288	0.735
3	....	0.767	0.454	0.810	0.611	0.697	....
				RT = 243° F.			
0	0.907	1.334	1.355	1.345	2.272	1.533	0.759
1	1.235	1.401	1.307	1.353	1.615	1.890	1.409
2	....	1.442	2.082	1.363	1.592	2.123	1.145
3	....	0.821	0.997	1.032	0.903	0.643	....
				RT = 261° F.			
0	0.829	1.622	1.729	1.629	2.339	1.673	1.209
1	1.320	1.886	1.255	1.724	2.514	1.923	1.572
2	....	1.605	1.658	2.230	1.859	1.456	1.670
3	....	0.814	1.056	1.087	1.165	0.554	....
				RT = 279° F.			
0	1.580	1.348	1.620	1.453	2.022	1.473	0.974
1	1.564	1.613	2.813	1.374	1.413	1.353	1.095
2	....	1.294	1.601	2.109	1.434	1.449	1.523
3	....	0.816	1.130	0.964	1.059	0.937	....
				RT = 297° F.			
0	1.575	2.241	1.465	1.445	1.414	1.310	1.005
1	1.101	1.708	1.930	1.321	2.081	2.390	1.300
2	....	1.112	1.241	1.521	1.336	1.231	1.106
3	....	1.110	1.229	1.167	1.322	1.351	....
				RT = 315° F.			
0	4.091	1.541	1.085	1.286	1.793	1.324	1.256
1	1.552	2.141	1.689	1.248	1.527	1.600	1.232
2	....	1.209	1.211	1.261	1.260	1.304	1.594
3	....	1.352	0.917	0.853	0.853	0.663	....

Conditions. The conditions during the experimental runs were maintained as follows:

(a) times and temperatures were used according to the schedule in the experimental design;

(b) raw material was prepared and used as indicated under technique above; and the head space in the can was negligible when packing;

(c) method of pack used was cold pack under 25″ of vacuum;

(d) meat before packing was stored at 32° F. for not longer than a four-day period, and after sealing of meat in cans, the cans were kept in a 50-60° F. water bath to assume uniform temperature distribution within the can;

(e) cooling water temperature was between 50-70° F;

(f) temperatures at 25 locations in the can were determined. The distribution of points is shown in Figure 1. The length of the thermocouples used was always greater than or equal to the radius of the can. Thermocouples were always inserted from the side wall of the container.

RESULTS AND DISCUSSION

Heating phase. The theoretical equation for the heating phase ($0<t<t_1$) is of the following form (from Eq. 1) in its first term approximation:

$$(RT - CT) = (RT - IT)\, A_{11}\, J_0(\mu_1 r) \sin \lambda_1 (z + l)\, e^{-k(\mu_1^2+\lambda_1^2)t} \qquad \text{Eq. 2a}$$

$$(RT - CT) = (RT - IT)\, a_{11}\, e^{-b't} \qquad \text{Eq. 2b}$$

It may be transformed into

$$\log (RT - CT) = \log [a_{11} (RT - IT)] - b_h t \qquad \text{Eq. 2c}$$

where $b_h = b' \log e$ is the slope of the heating curve on semi-log graph and a_{11} may be called the intercept coefficient of the curve and varies from location to location.

Regression equations were derived for all of the 25 locations tested by determination of the intercept and of the slope by the least squares method. The values of the calculated parameters were defined as $\log [\hat{a}_{11} (RT-IT)]$

TABLE 4

Lag coefficients ($C_1 = \hat{a}_{11}/a_{11}$) for six retort temperatures

Location	Retort Temperature (°F.)						
	225	243	267	279	297	315	Average
1	0.694	0.881	0.787	1.499	1.494	3.881	1.536
7	0.764	0.720	1.147	0.924	0.954	1.192	0.950
8	0.982	1.204	1.287	1.524	1.744	1.513	1.376
14	0.364	1.373	1.532	1.067	1.267	1.201	1.134
20	0.779	1.214	1.771	1.615	1.173	1.690	1.374
21	0.666	0.713	0.707	0.708	0.964	1.174	0.822
22	0.310	0.681	0.721	0.772	0.839	0.626	0.658
23	0.515	0.656	0.691	0.613	0.742	0.544	0.627
24	0.417	0.617	0.796	0.773	0.903	0.583	0.682
25	0.605	0.558	0.481	0.813	1.173	0.576	0.701
Average	0.610	0.860	0.992	1.031	1.125	1.298	0.986
2	0.738	0.892	1.085	0.902	1.499	1.031	1.024
6	0.603	1.025	1.119	0.985	0.876	0.886	0.916
9	0.739	0.963	1.296	1.109	1.174	1.471	1.125
13	1.112	1.299	1.322	0.931	1.643	1.100	1.235
15	0.580	1.079	1.200	0.968	0.832	0.904	0.927
16	0.793	1.225	0.976	0.942	0.730	0.713	0.897
17	0.694	0.747	1.222	1.156	0.833	0.691	0.891
18	0.846	0.937	1.094	0.844	0.796	0.742	0.875
19	0.963	1.588	1.088	1.884	0.921	0.975	1.237
Average	0.785	1.084	1.156	1.080	1.033	0.946	1.014
3	0.726	0.713	0.910	0.853	0.771	0.571	0.757
4	0.717	0.659	0.798	0.712	0.708	0.630	0.704
5	1.031	1.196	1.231	1.064	0.744	0.944	1.035
10	0.676	0.707	0.679	1.521	1.044	0.913	0.923
11	0.558	0.681	0.868	0.692	0.665	0.628	0.682
12	0.809	0.873	1.360	0.764	1.125	0.826	0.960
Average	0.753	0.803	0.974	0.934	0.843	0.752	0.843
Grand Average	0.712	0.900	1.021	0.990	0.985	0.985	0.952

and $\hat{b}_h$ respectively. The regressions were computed for the average experimental values of log (RT − CT) for 9 curves (3 curves for each of the 3 cuts of the round of beef). The readings at t = 0 were disregarded and regression computations were made only for the linear part of the curve observed from the plot, thus eliminating the possible influence of thermal lag at the beginning of the heating on the value of the slope.

The time of processing (t_1) was not equal for all of the 9 curves which resulted in unequal weights for the averages used in the regression. The equations thus derived approximate the experimental data best for shorter processing times. Since the experiment was planned for investigation of high temperature processes which give sufficient sterilizing effect for relatively short times, this method was found satisfactory for the useful range of processing times at the temperatures used.

It was also observed that the slope values computed for very short processes were relatively smaller than those for longer runs, thus reflecting the

TABLE 5

Distribution of lag coefficients (C_1) in 300 x 308 can

r (cm)	z (cm)						
	2.75	2.00	1.00	0.00	−1.00	−2.00	−2.75
				RT = 225° F.			
0	0.694	0.738	0.726	0.717	1.031	0.603	0.764
1	0.982	0.739	0.676	0.558	0.809	1.112	0.364
2	....	0.580	0.793	0.694	0.846	0.963	0.779
3	....	0.666	0.310	0.515	0.417	0.605	....
				RT = 243° F.			
0	0.861	0.892	0.713	0.658	1.196	1.025	0.720
1	1.204	0.963	0.707	0.681	0.873	1.299	1.373
2	....	1.079	1.225	0.747	0.937	1.588	1.214
3	....	0.713	0.681	0.656	0.617	0.558	....
				RT = 261° F.			
0	0.787	1.085	0.910	0.798	1.231	1.119	0.147
1	1.287	1.296	0.679	0.868	1.360	1.322	1.532
2	....	1.200	0.976	1.222	1.094	1.089	1.771
3	....	0.707	0.721	0.691	0.796	0.481	....
				RT = 279° F.			
0	1.499	0.902	0.853	0.712	1.064	0.985	0.924
1	1.524	1.109	1.521	0.692	0.764	0.931	1.067
2	....	0.968	0.942	1.156	0.844	1.084	1.615
3	....	0.708	0.772	0.613	0.723	0.813	....
				RT = 297° F.			
0	1.494	1.499	0.771	0.707	0.774	0.876	0.954
1	1.073	1.174	1.044	0.665	1.125	1.643	1.267
2	....	0.832	0.730	0.833	0.786	0.921	1.173
3	....	0.964	0.839	0.742	0.903	1.173	....
				RT = 315° F.			
0	3.881	1.031	0.571	0.630	0.944	0.886	1.192
1	1.515	1.471	0.913	0.628	0.826	1.100	1.201
2	....	0.904	0.713	0.691	0.742	0.975	1.690
3	....	0.906	0.702	0.296	1.213	0.423	....

influence of the initial thermal lag period which does not properly belong to the linear portion of the heating curve. A separate regression calculation for each heating curve could have introduced bias in determination of the constants of the equations and resulted in less reliable estimates than those afforded by the procedure adopted above. The estimates of variance obtained must be considered as estimates of variation of means of nine determinations.

The values of $\hat{a}_{11}$ obtained at all the locations and temperatures have been tabulated in Tables 2 and 3. The lag coefficient (Tables 4 and 5) (or correction factor for the theoretically expected a_{11}) was evaluated as $C_1 = \hat{a}_{11}/a_{11}$ and may be used in connection with Equation 2.

The values of the slopes $\hat{b}_h$ for all the processing conditions encountered during this investigation were found to be substantially the same for all locations in the can but varying with the processing temperature. The aver-

TABLE 6

Slopes ($\hat{b}_h$) of heating curves and their variation

RT (°F.)	225	243	261	279	297	315	Average
$\hat{b}_h \times 10^3$ (min.)$^{-1}$	23.75	27.29	30.27	29.58	29.62	26.24	27.79
C.V. ($\hat{b}_h$) (%)	7.72	4.57	5.69	5.06	6.02	9.32	6.40
C.V. (RT − CT) (%)	33.16	18.44	21.00	16.79	17.63	11.66	19.78
C.V. (RT − IT)a_{11} (%)	10.96	5.72	7.80	5.51	6.10	4.78	6.81

age values of $\hat{b}_h$ are given in Table 6 for the 6 retort temperatures and may be used in connection with Equation 2 (c) with the corrected a_{11} to determine the temperatures at any location in the can for the retort temperature range 225-315°F. The coefficient of variation of $\hat{b}_h$ is given in Table 6 and did not exceed 9.32%. In the same table the standard error of estimate is given in the determination of log (RT−CT). This error was expressed as percentage of (RT−CT) (*17*, p. 248) and its average was 19.78%. Table 6 indicated better approximation of the data by the regression equations for higher retort temperatures.

The heating equation describing the thermal histories of the can contents is as follows:

$$(RT - CT) = (RT - IT)\, C_1\, a_{11}\, e^{-\hat{k}(\mu_1^2+\lambda_1^2)t} \qquad \text{Eq. 3a}$$

or with average values of the factors determined during this investigation for 300 x 308 can

$$(RT - CT) = (RT - IT)\, 0.934\, a_{11}\, e^{-0.06515t} \qquad \text{Eq. 3b}$$

The departures from the theoretical expectation will be reflected best if the values for C_1 are taken from Table 4 for each location and temperature separately, with the average value of $\hat{k}$ (Fig. 2) being taken from the curve representing k = f (RT).

As a general approximation of the thermal history of the can contents it will be sufficient to use Equation 3b with the average $\hat{k}$ and C_1 given in Table 7. This table also indicates the average coefficients of variation for these characteristics.

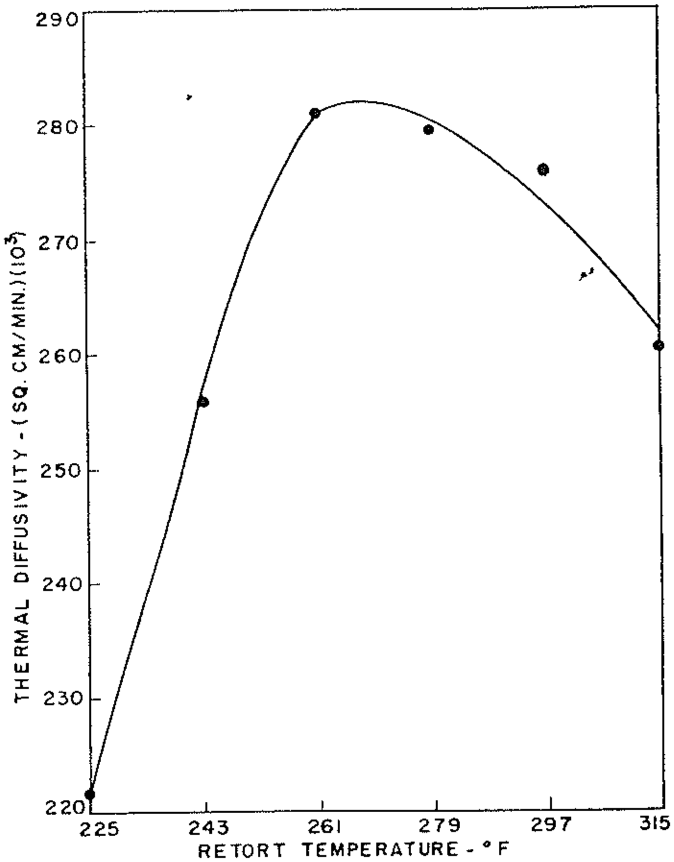

Figure 2. Relationship between thermal diffusivity and retort temperature.

Cooling phase. The exact analytical solution of the time-temperature relationship for the cooling phase (Eq. 1) in its first term approximation may be transformed as follows:

$$\mathrm{CT} = \mathrm{RT} - (\mathrm{RT} - \mathrm{CT})_{t_1} + (\mathrm{TC} - \mathrm{RT}) - (\mathrm{TC} - \mathrm{RT})\, a_{11}\, e^{-b'(t-t_1)} \qquad \text{Eq. 4a}$$

$$\mathrm{CT} = -(\mathrm{RT} - \mathrm{CT})_{t_1} + \mathrm{TC} - (\mathrm{TC} - \mathrm{RT})\, a_{11}\, e^{-b'(t-t_1)} \qquad \text{Eq. 4b}$$

$$(\mathrm{CT} - \mathrm{TC}) + (\mathrm{RT} - \mathrm{CT})_{t_1} = (\mathrm{RT} - \mathrm{TC})\, a_{11}\, e^{-b'(t-t_1)} \qquad \text{Eq. 4c}$$

TABLE 7

Average coefficients and parameters to be used with prediction equation for heating phase (Eq. 3a) and their variation

RT (°F.)	225	243	261	279	297	315	Average
$\hat{k} \times 10^3$ (sq. cm/min.)	221.63	255.80	281.08	279.46	275.79	259.97	262.29
C.V. ($\hat{k}$) (%)	7.72	4.57	3.69	5.06	6.02	9.32	6.40
C.V. ($\hat{a}_{11}$) (%)	10.46	5.86	6.60	5.47	4.56	5.28	6.37
$C_1 = \hat{a}_{11}/a_{11}$	0.708	0.895	1.020	0.992	0.990	0.296	0.934

Since $RT - TC = [(CT_{t_1} - TC) + (RT - CT)_{t_1}]$ or $= (m + g)$ and $b' = \frac{1}{f_h \log e}$ as well as $(t-t_1) = x_{cl}$, in Ball's terminology the equation may be expressed as:

$$(CT-TC+g) = (CT_{t_1}-TC+g)\, a_{11}\, e^{-b'(t-t_1)} = (m+g)\, a_{11}\, e^{(x_{cl}/f_h)} \qquad \text{Eq. 4d}$$

and compared with the expression used by Ball *(2)* to approximate the cooling curves of the form:

$$(CT - TC) = j\,(CT_{t_1} - TC)\, e^{-b''(t-t_1)} = mje^{(x_{cl}/f_{cl})} \qquad \text{Eq. 5a}$$

in which he assumed $f_{cl} = f_h$
if the exact equation (Eq. 4d) is solved for $(CT-TC)$ one obtains

$$\begin{aligned}(CT - TC) &= a_{11}\, me^{x_{cl}/f_h} + a_{11}\, g\, e^{x_{cl}/f_h} - g\\ &= a_{11}\, me^{x_{cl}/f_h} + g\,(a_{11}\, e^{x_{cl}/f_h} - 1)\end{aligned} \qquad \text{Eq. 5b}$$

If Eq. 5a and Eq. 5b are compared at the beginning of cooling when $(t-t_1) = x_{cl} = 0$ one obtains

$$j = a_{11} + \frac{g}{m}(a_{11} - 1) \qquad \text{Eq. 6a}$$

which conclusively proves that j is not equal to a_{11} at the center of the container as used for cooling curves. This points out the inconsistency of using j determined from heating curves at can center for application to the cooling curves since there $j = A_{11} = a_{11}$ by its definition. This was to be expected since j determines the intercept on y-axis when $(t - t_1) = x_{cl} = 0$ for the $(CT - TC) = f\,(t)$ plot on semi-log paper and a_{11} determines the intercept when $(t - t_1) = 0$ for the $[(CT - TC) + g] = f\,(t)$ plot. Thus j was found to be a function of g/m ratio and greater than a_{11} in all locations in the container with the exception of locations where $a_{11} = 1$.

At any time of cooling j may be determined from the equality of the equations 5a and 5b with $E_{cl} = e^{(x_{cl}/f_{cl})}$ and $E_h = e^{x_{cl}/f_h}$ as

$$j = a_{11} + \frac{g}{mE_{cl}}(a_{11}\, E_h - 1) \qquad \text{Eq. 6b}$$

The above relationship solved for E_{cl} with the value of j obtained from Eq. 6a.

$$E_{cl} = \frac{(a_{11}\, E_h - 1)}{(a_{11} - 1)} = e^{x_{cl}/f_{cl}} = \frac{(a_{11}\, e^{x_{cl}/f_h} - 1)}{(a_{11} - 1)}$$

indicates that f_{cl} is a function of a_{11} and f_h but cannot be equal to f_h as assumed by Ball. If the assumption is made that $f_{cl} = f_h$ then $E_{cl} = E_h = 1$, which can occur only when $(t-t_1) = x_{cl} = 0$ and at no other time during the cooling phase. The analysis of variance made for $\hat{f}_h$ and $\hat{f}_{cl}$ values determined by regression methods confirmed *(10)* these theoretical expectations and showed in addition that $\hat{f}_{cl}$ and $\hat{f}_h$ values are not affected by either processing temperature or location in the can.

The cooling curves were evaluated by regression method in the form of

$$\log\,(CT - TC) = \log\,[j\,(CT_{t_1} - TC)] - b_{cl}\,(t - t_1) \qquad \text{Eq. 5c}$$

where $b_{cl} = b'' \log e$

TABLE 8

Average coefficients and parameters to be used with prediction equations for the cooling phase (Eqs. 3a, 3b, and 4b) and their variation

RT (°F.)	225	243	261	279	297	315	Average
$\hat{b}_{cl} \times 10$ (min.)$^{-1}$	26.95	25.22	25.97	24.52	25.28	24.91	25.48
C.V. ($\hat{b}_{cl}$) (%)	12.31	19.02	42.53	36.47	25.33	19.26	25.82
C.V. (CT − TC) (%)	6.08	9.33	17.83	13.38	10.72	8.00	10.90
C.V. (CT_{t_1} − TC)$\hat{j}$ (%)	7.35	3.46	23.96	18.41	14.04	9.64	12.81
$C_2 = \hat{a}'_{11}/\hat{a}_{11}$	1.329	0.839	0.848	0.373	0.865	0.893	0.941
$C_3 = \hat{j}/j$	0.873	0.730	0.802	0.873	0.867	0.755	0.817

in the same manner as the heating curves. The regression computations, however, were made on the average of three curves (for the three cuts of beef) instead of nine, since the value of $\hat{j}$ was found to be dependent on the g/m ratio. Thus the average was taken from three curves which displayed approximately the same g/m values. Again the linear portion of the curves was estimated from their semi-log plot to exclude the data from the intermediate cooling phase.

The regression calculations yielded values for the slopes of the cooling curves ($\hat{b}_{cl}$) (Table 8) and for the values of the cooling curve intercept coefficient ($\hat{j}$). Since the slope $\hat{b}_{cl}$ was found not to be affected by the temperatures or locations *(10)* its average (0.025475) may be used in connection with Equation 5(b) for all conditions encountered in this investigation. The computed values of $\hat{j}$ were compared with the theoretical j and with $\hat{j}_1$ derived by means of Equation 6(a) using $\hat{a}_{11}$ determined from heating equations. By means of the same equation $\hat{a}'_{11}$ was computed from

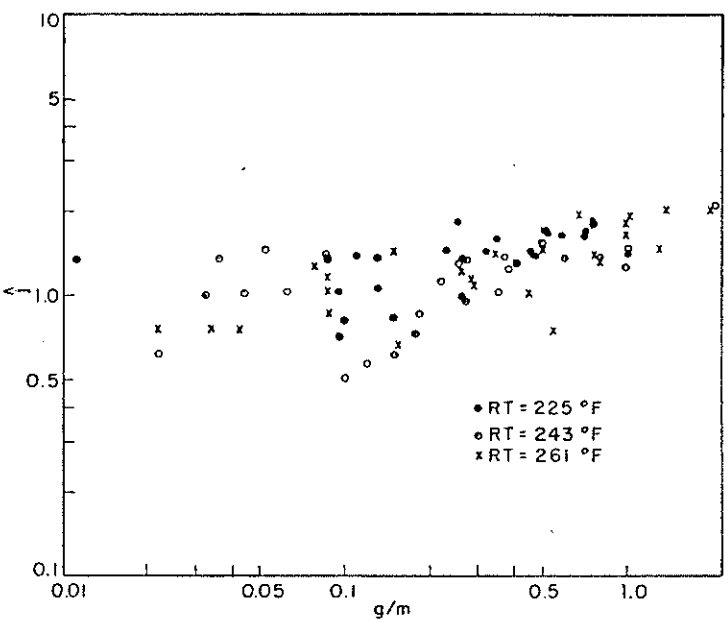

Figure 3. Scatter diagram in log-log coordinates for the $\hat{j}$ and g/m relationship at low retort temperatures.

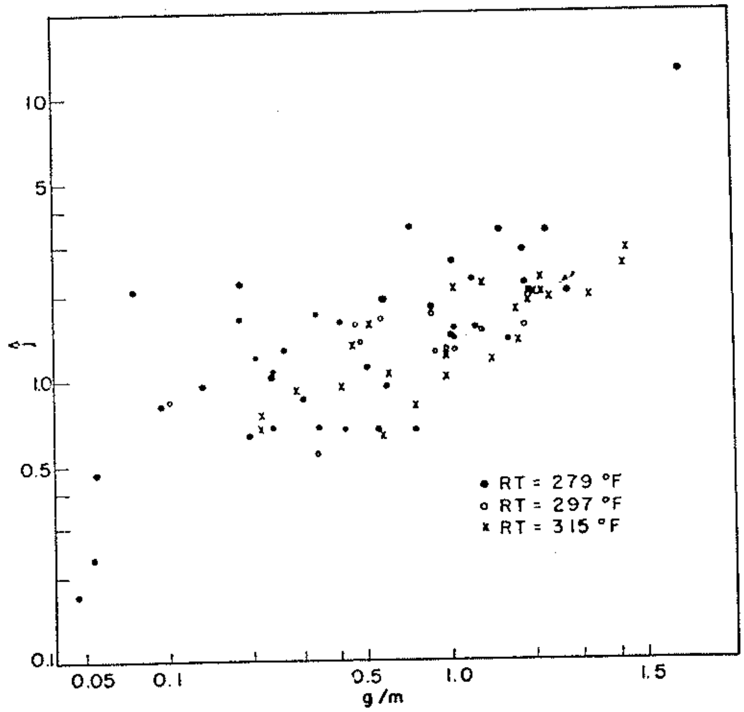

Figure 4. Scatter diagram in log-log coordinates for the j and g/m relationship at high retort temperature.

$\hat{j}$ values and the ratio of $\hat{a}'_{11}/\hat{a}_{11} = C_2$ was determined for all the processing conditions at all of the locations in the can. This ratio was found substantially constant and may be used to determine the $\hat{j}$ if only heating data are available by means of Equation 6(a). The average value of C_2 was found to be 0.941. Its values for the six retort temperatures are given in Table 8. The same table gives the values of $\hat{j}/j = C_3$ ratio which also was found substantially constant. This ratio may serve as a correction factor for the theoretically expected j and is equal to 0.817. The computed $\hat{j}$ were plotted as a function of g/m and confirmed the theoretical expectation expressed in Equation 6(a) that $\hat{j}$ is a function of g/m. (Figs. 3 and 4.) The plot resulted in an approximately straight line in log-log coordinates. The slope seemed to increase with the increase in retort temperature.

In view of the apparent constancy of the C_2 ratio and dependency of $\hat{j}$ on g/m, it would seem to be advantageous to use as a prediction equation for the cooling phase Equation 4(c) and to plot the log $(CT - TC + g)$ against $(t - t_1)$ rather than to plot the log $(CT - TC)$ as it is done at the present time. This procedure would save separate determinations of $\hat{f}_h$ and $\hat{f}_{cl}$ and of computing $\hat{j}$; also $\hat{a}'_{11}$ could be estimated from heating data by means of C_2 ratio to yield the following prediction equation for the cooling period.

$$(CT - TC) + (RT - CT)_{t_1} = (RT - TC)\, C_2\, \hat{a}_{11}\, e^{-\hat{k}(\mu_1^2+\lambda_1^2)(t-t_1)} \quad \text{for } (t - t_1) > 10 \qquad \text{Eq. 7a}$$

with $\hat{a}_{11}$ estimated from heating data. This equation would take into account all departures from theory during the heating period. Equation 7(a) combined with the prediction equation for the heating period (Eq. 3a) would give a general relationship of the following form:

$$(CT-TC)+(RT-CT)_{t_1}=(RT-TC)\ C_2\ C_1\ a_{11}\ e^{-\hat{k}(\mu^2_1+\lambda^2_1)(t-t_1)} \qquad \text{Eq. 7b}$$

The factors in the above equation can be computed either on the basis of theoretical considerations (μ^2_1; λ^2_1; a_{11}) or from the results of this investigation (C_1; C_2 and $\hat{k}$). The average factor $C_2=0.941$ for all temperatures, and the factor C_1 to be used may be taken from Table 7; if only an approximate relationship is found sufficient, the average $C_1=0.934$ may be used. The values of $\hat{k}$-thermal diffusivity as in the case of heating equation may be taken from Fig. 2 for corresponding retort temperatures or an average value of $\hat{k}=0.2623$ sq cm/min may be used. Thus the average time-temperature relationship during cooling phase for 300 x 308 cans with beef is as follows:

$$(CT - TC) + (RT - CT)_{t_1} = (RT - TC)\ 0.879a_{11}\ e^{-0.06515(t-t_1)}$$
$$\text{for } (t - t_1) > 10 \qquad \text{Eq. 8a}$$

The alternate equation of the form used by Ball would be

$$(CT - TC) = (CT_{t_1} - TC)\ C_3\, je^{-0.05563(t-t_1)} \text{ for } (t - t_1) > 10 \qquad \text{Eq. 8b}$$

and where j is computed from Eq. 6(a) and average $C_3=0.817$.

The average variation of the log [(CT−TC)] as determined from regression of means of three curves is shown in Table 8, and the average C.V. was 10.90% of (CT−TC). The average coefficients of variation of $\hat{b}_{cl}$ are presented in Table 8; and the average C.V. was 25.82%, considerably larger than the variation in $\hat{b}_h$. The average C.V. of $\hat{j}$ $(CT_{t_1}-TC)$ was found to be 12.81%. The larger variation in $\hat{b}_{cl}$ was reflected also in the analysis of variance of $\hat{f}_{cl}$ *(5)*.

SUMMARY

An investigation of time, temperature, and space relationship during canning of beef extended to high retort temperatures (up to 315°F) was made. Thermal histories were obtained for 25 locations in 300 x 308 cans packed with round of beef. The functional relationships were determined by regression methods for the heating and cooling phases of processing with the exception of first ten minutes of the cooling ("intermediate" cooling). Physical properties of beef not available in the literature have been ascertained.

In addition to the establishment of empirical relationships by means of the determination of correction factors for the theoretical relationship for the whole space of the can (not only the geometrical center) for both heating and cooling phases, a method has been devised to obtain empirical constants for the cooling phase from experimental data gathered during heating. The experimental work thus has been reduced for future research with other commodities. The precision of results has been evaluated by the estimate of experimental errors associated with them.

On the basis of the experimental results obtained during the investiga-

tion and for the processing conditions prevailing during the experiment the following conclusions can be drawn:

The average time-temperature relationship existing in the can is expressed best during the heating phase of thermal process ($t<t_1$) by the expression below:

$$(RT - CT) = (RT - IT)\, 0.934\, a_{11}\, e^{-0.06515t}$$

and during the cooling phase ($t - t_1 > 10$) by the following expression:

$$(CT - TC) + (RT - CT)_{t_1} = (RT - TC)\, 0.879\, a_{11}\, e^{-0.06515(t-t_1)}$$

The availability and manner of determination of the above empirical time-temperature-space relationship should lead to improvement of process calculations and to the increase in their precision and reliability. Ultimately it should become a part of the overall correlation of fundamental laws governing the process variables and thus contribute to the development of not only safe but also palatable products.

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