

A Long-Pulse Method of Determining Thermal Diffusivity

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The solution of the one-dimensional heat-flow equation for an infinite insulated slab, heated by a long thermal-radiation pulse, showed that the thermal diffusivity of the slab could be determined from its thickness and the time necessary for the rear surface to attain one-half the maximum temperature. The Army Natick Laboratories solar furnace provided the thermal flux, and the rear surface temperature function of the 1100-F (99 percent pure) aluminum sample was measured with a Barnes radiometer. The thermal diffusivity values of four samples, 1, 2, 3, and 4 cm thick are 0.747, 0.773, 0.714 and 0.759 sq cm per sec with standard deviations of 0.006, 0.020, 0.019, and 0.024 respectively. The overall average thermal diffusivity is 0.75 sq cm per sec, which is within the range of values found by other investigators for similar materials. The individual sample measurements varied from their mean only about two percent. Heat loss from the sample under these conditions is negligible.

RECENT advances in the use of high energy devices have increased the necessity of knowing the thermal properties during high rates of heating. Often thermal diffusivity, $\alpha = k/\rho c$, has been calculated from individual values of the conductivity, k , heat capacity, c , and density, ρ , obtained in steady-state experiments. In general, such experiments were time consuming and required rigid control of the environment. Sidles and Danielson,¹ for instance, reported that their data taking required 15 minutes and that the ambient temperature had to be held within $\frac{1}{4}$ deg C for this period.

There is some advantage in determining diffusivity directly. Methods for doing this have been employed based upon solution of the general heat-flow equation with various boundary conditions. The experimenter usually finds the time-temperature relationship at some known depth in the sample being heated, and calculates the diffusivity from this data.

Woisard² has developed a short pulse (five-millisecond) method in which the data are taken within ten seconds. An electrical pulse heats a thin disk bonded between two essentially infinitely long rods. The diffusivity of the rods is determined by finding the time necessary for the temperature to rise to a certain percentage of its maximum. His results agreed very closely with those obtained by Sidles and Danielson¹ for a similar material.

Butler and Inn³ measured diffusivity using a long pulse of thermal radiation to heat the material. Thermocouples measured the temperature rise at two known depths in the sample. The diffusivity was calculated from the slope of the temperature function and the temperature difference between the thermocouples.

Parker et al⁴ have used a xenon flash lamp with a thermocouple at the rear of the sample. A measurement at the time necessary for the rear surface to come to one-half its maximum temperature gives the data necessary to calculate the diffusivity.

Cabannes⁵ used thermopiles to measure the phase lag between a sinusoidally varying incident beam of thermal radiation and the varying intensity of the emitted infrared radiation at the rear of his sample. The thermal diffusivity is inversely proportional to this phase lag.

The method presented here contains several features adapted from these previous works. A sample of finite length is heated by a long thermal pulse of constant irradiance while a radiometer measures the temperature rise at the rear surface. The time necessary for the temperature to rise to one-half its maximum is measured and the diffusivity is calculated from this time, the pulse length, and the sample thickness. Significant readings are produced within a few seconds so that corrections for sample cooling are small. For the samples studied here the loss is negligible. Also, it is not necessary to control the ambient temperature closely. The temperature gradient within the sample can be kept small, making the method applicable to coated samples or those whose surface might be damaged by the high temperatures produced using an instantaneous-pulse method.

NOMENCLATURE

a —Absorptivity
α —Thermal diffusivity ($\text{cm}^2 \text{sec}^{-1}$)
c —Heat capacity ($\text{cal gm}^{-1} \text{ }^\circ\text{C}^{-1}$)
H —Irradiance ($\text{cal cm}^{-2} \text{sec}^{-1}$)
k —Thermal conductivity ($\text{cal cm}^{-1} \text{sec}^{-1} \text{ }^\circ\text{C}^{-1}$)
L —Length (cm)
n —Positive integer from 1 to ∞
ρ —Density (gm cm^{-3})
t —Time (sec)
$t_{\frac{1}{2}}$ —Time required for temperature to reach one-half its maximum (sec)
τ —Exposure duration (sec)
w —Temperature rise ($^\circ\text{C}$)
w_m —Maximum temperature rise
w_r —Relative temperature rise

Theory and Experimental Requirements

The temperature rise in a slab, irradiated on one face, with the other face insulated, can be calculated from a solution of the basic heat-flow equation.⁶

$$k \frac{\partial^2 w}{\partial x^2} = \rho c \frac{\partial w}{\partial t}$$

or

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{\alpha} \frac{\partial w}{\partial t} \quad (1)$$

where w is the temperature rise at depth x and t is the time.

Equation (1) can be solved analytically for certain ideal conditions, i.e., an inert, homogeneous, opaque slab, with constant thermal properties over the range of temperature of interest. One-dimensional heat flow can be arranged by irradiating the face uniformly and preventing lateral flow, for example, by insulating the sides and for this condition, if the irradiance at the front face, $x = L$, is H , and the absorptance of that surface is a , $a = 1 - r$, where r is the reflectance, neglecting losses from the front or back face, $x = 0$, the boundary conditions are:

$$\begin{aligned} w &= 0 \text{ at } t = 0 \text{ for all } x \\ \partial w / \partial x &= 0 \text{ at } x = 0 \text{ for all } t \\ k(\partial w / \partial x) &= aH \text{ at } x = L \text{ for } t > 0 \end{aligned} \quad (2)$$

The solution for a step pulse as given by Carslaw and Jaeger⁶ is

$$w = \frac{aHL}{k} \left[\frac{\alpha t}{L^2} + \frac{3x^2 - L^2}{6L^2} - \frac{2}{\pi^2} \sum \frac{(-1)^n}{n^2} \cdot e^{-(n^2 \pi^2 \alpha / L^2)t} \cos \frac{n\pi x}{L} \right] \quad (3)$$

and for $x = 0$,

$$w = \frac{aH}{k} \left[\frac{\alpha t}{L^2} - \frac{1}{6} - \frac{2}{\pi^2} \sum \frac{(-1)^n}{n^2} e^{-(n^2 \pi^2 \alpha / L^2)t} \right] \quad (4)$$

As shown in the Appendix, the series term accounts for less than 1 percent of the total w if $\alpha t / L^2 > 0.44$ and for this condition

$$w = \frac{aHL}{k} \left[\frac{\alpha t}{L^2} - \frac{1}{6} \right] \quad (5)$$

For a rectangular pulse of duration τ , Eq. (3) becomes

$$w = \frac{aHL}{k} \left[\frac{\alpha \tau}{L^2} - \frac{2}{\pi^2} \sum \frac{(-1)^n}{n^2} e^{-(n^2 \pi^2 \alpha / L^2)t} \cdot (1 - e^{-(n^2 \pi^2 \alpha \tau) / L^2}) \cos \frac{n\pi x}{L} \right] \quad (6)$$

and for $x = 0$

$$w = \frac{aHL}{k} \left[\frac{\alpha \tau}{L^2} - \frac{2}{\pi^2} \sum \frac{(-1)^n}{n^2} e^{-(n^2 \pi^2 \alpha / L^2)t} (1 - e^{-(n^2 \pi^2 \alpha \tau) / L^2}) \right] \quad (7)$$

From considerations of the absorbed energy, $aH\tau$, and the heat capacity, ρcL , or from Eq. (7) at long times when only the first term is significant, the maximum temperature attained by the back surface, w_m is

$$w_m = \frac{aH}{\rho cL} = \frac{aH\alpha \tau}{kL} \quad (8)$$

The temperature rise of this surface during the rectangular pulse, relative to the maximum temperature rise, which occurs after the pulse, is

$$w_r = \frac{w}{w_m} = \frac{\frac{aHL}{k} \left[\frac{\alpha t}{L^2} - \frac{1}{6} \right]}{\frac{aH\alpha \tau}{kL}} = \frac{t}{\tau} - \frac{L^2}{6\alpha \tau} \quad (9)$$

or

$$\alpha = \frac{L^2}{6} \left(\frac{1}{t - w_r \tau} \right) \quad (10)$$

Defining a half time, $t_{\frac{1}{2}}$, as the time for the back surface to reach half its maximum temperature, i.e., $w_r = \frac{1}{2}$,

$$\alpha = \frac{L^2}{6} \left(\frac{1}{t_{\frac{1}{2}} - \frac{1}{2}\tau} \right) \quad (11)$$

The diffusivity can be determined then from the slab thickness, the pulse duration and the half time. It is not necessary to know the temperature rise absolutely; relative values are sufficient. It is necessary that the emissivity be constant during the exposure and also that the irradiance and absorptance remain constant. These and the other requirements indicated above can be met as discussed in the next section.

The above discussion assumes no loss of energy during the exposure. There will be some loss at both faces by radiation and convection and also from the sides by these or other processes. This loss could be taken into account by using solutions of the basic heat-flow equation with different boundary conditions but this becomes rather complicated and in any case the appropriate loss constants are not well known. Accordingly, another procedure was used to estimate the effect of this loss.

The temperature of the front surface is higher than that of the back surface by an amount $aHL/2k$, except for very short times, and the loss from that face will be correspondingly greater. The total loss will reduce both the temperature of the back surface during the ex-

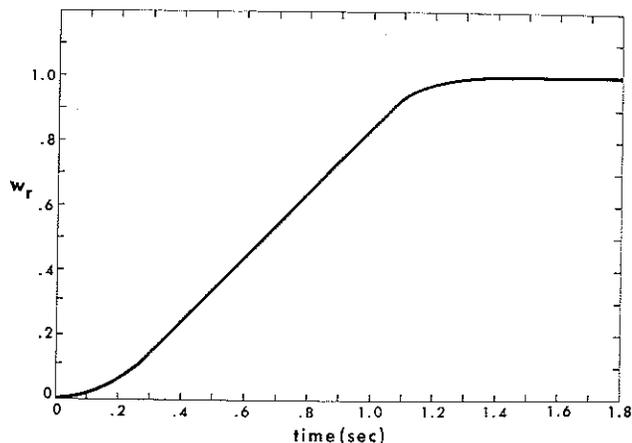


Fig. 1—Calculated time-relative temperature function.

posure and the maximum temperature it can attain and since α depends on the ratio of these two factors, the effect of these losses on α will be less than on either temperature.

For $t > \tau$, the back surface will gain heat from the rest of the sample. From that effect, the temperature will tend to increase. Also, the back surface will lose heat to the surroundings. From that effect the temperature will tend to decrease. The net effect will depend on the difference between the two effects. If it can be shown that the expected temperature rise from the first effect is small and that the overall change is also small, then obviously the effect of heat loss is small.

A calculated temperature record for the back surface, obtained as described in Eq. (7) ($\tau = 1$), is shown in Fig. 1. The net temperature change after exposure is small, i.e., the curve is essentially flat. The relative temperature for the sample was calculated for $t > \tau$ using Eq. (7) and the following factors: $aH = 12$ cal per sq cm per sec, $L = 1$ cm; α , from Eq. (11) without correction for heat loss, = 0.75 sq cm per sec, $k = 0.48$ cal per cm per sec per deg C, $\tau = 1$ sec. At $t = 1.5$ sec, w is within 0.025 deg C or 0.1 percent of the maximum temperature; at $t = 2$ sec, w is within 0.00002 deg C of the maximum or less than 0.0001 percent. Estimation of the rate of cooling shown in Fig. 2 and from similar curves over the first ten seconds is about 0.01 deg C per deg C per sec. Therefore, even for times as long as ten seconds this is negligible and indicates the effect of such cooling on α is probably less than 0.1 percent. For longer exposures, the effect of the loss may be slightly greater, but for the present work no account was taken of the loss in calculating α .

Experimental Procedure

The Army Natick Laboratories solar furnace described by Cotton et al.⁷ provided the thermal flux used in these experiments. The furnace produces an image of the sun about 10 cm in diameter; the center 2.5 cm diameter receives a flux that is uniform within 5 percent

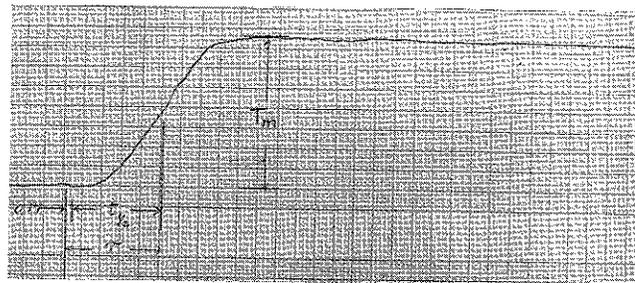


Fig. 2—Signal trace, sample 1-A.

of the average over this area. The samples were right circular cylinders, 2.5 cm diameter in order to receive this uniform flux over the entire front surface.

A shutter system allows the exposure duration to be measured to 0.01 sec and a short-circuiting switch activates a timing pulse at the recorder so that both the starting time and duration can be measured. There is a slight delay (0.14 sec) due to the lag in response of the hydraulically controlled shutters. This was measured and taken into account in all calculations.

A Barnes radiometer, Model R8B1, placed about 10 feet behind the sample, measured the infrared radiation emitted by the rear surface, over the center 0.5 inch diameter. Since the temperature rise observed was less than 20 deg C above ambient, it was assumed that the radiometer output was proportional to the temperature rise.

The radiometer has an adjustable bandwidth control but for this work it was always set for a time constant of 0.016 sec. The recorder was a Sanborn 152-100B.

The samples were sections of Alcoa 1100-F, 1 inch diameter and cut 1, 2, 3, and 4 cm long. Presumably they are 99 percent pure aluminum; one assay showed 0.4 percent iron and silicon present. No assay was made of the samples used in this work.

The samples were annealed at 350 deg C for eight hours and cooled slowly over night to eliminate some effect of previous thermal history. Both front and rear surfaces were coated with Parson's Optical Black primer and paint. They were mounted in marinite blocks covered with a heavy aluminum tape. The blocks were mounted behind a water-cooled shield so that only the front surface of the sample was exposed to the beam.

The exposure duration and radiance were varied to produce the same temperature rise in all samples.

The data in Table I show the results and variations. There is no consistent variation with length, and all averages agree with a probable error of less than 3 percent.

Discussion

This method of determining thermal diffusivity is rapid and direct, and requires a minimum number of approximations. The flux must be constant and uniform over the sample face and remain so for a relatively

TABLE 1—EXPERIMENTAL RESULTS OBTAINED FROM Al 1100F SAMPLES

Sample 1 (1 cm)				
Exp. No.	T_m^*	$t_{1/2}$ (sec)	τ (sec)	α (cm ² /sec)
1	2.6	0.715	0.99	0.758
2	4.4	0.725	0.99	0.725
3	4.2	0.720	1.00	0.758
4	5.5	0.720	0.99	0.741
5	4.5	0.715	0.99	0.758
6	4.7	0.715	0.98	0.741
				0.747 (av)
				$\sigma = 0.006$
Sample 2 (2 cm)				
1	3.9	2.89	3.96	0.733
2	5.8	2.85	3.99	0.780
3	5.6	2.84	3.97	0.780
4	6.1	2.85	3.98	0.775
5	6.1	2.84	3.95	0.771
6	5.7	2.84	3.99	0.790
				0.773 (av)
				$\sigma = 0.020$
Sample 3 (3 cm)				
1	6.7	6.50	9.05	0.759
2	6.7	6.61	8.96	0.704
3	7.2	6.59	9.04	0.725
4	7.7	6.65	9.03	0.703
5	7.5	6.63	9.01	0.706
6	5.5	6.65	9.02	0.701
7	4.7	6.66	9.03	0.699
				0.714 (av)
				$\sigma = 0.019$
Sample 4 (4 cm)				
1	4.6	11.78	16.09	0.714
2	6.4	11.50	16.20	0.784
3	6.6	11.67	16.11	0.738
4	6.1	11.43	16.00	0.777
5	6.5	11.64	16.17	0.750
6	6.0	11.33	15.90	0.784
				0.759 (av)
				$\sigma = 0.024$

* T_m is the temperature rise measured as cm of trace deflection in the chart.

short time. The radiometer measures the time-temperature function without disturbing the heat flow within the sample.

The average temperature rise was only 20 deg C and the front surface of Sample 1 only ran to 22 deg C; all others were lower. The flux density ranged from 12 to 3 cal per sq cm per sec and exposure times from 1 to 16 seconds. Under these conditions most samples are not subjected to extreme rates of heating, which might affect their properties.

Several values of the diffusivity of this material are reported in the literature. A typical example is that reported by Butler and Inn³. At about the same tempera-

ture they found that the diffusivity of Al 1100-F changed from about 0.9 sq cm per sec to 0.84 sq cm per sec depending upon the prior thermal history of the sample. Lucks and Deem⁸ reported a change from 0.52 sq cm per sec to 0.91 sq cm per sec for a sample of Al 1100-F after it had been heated to 575 deg F and cooled. It seems reasonable that the average value of 0.75 sq cm per sec found in these experiments is the correct value for these samples that had been heat treated.

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APPENDIX

The criteria used here to determine that the exponential term in Eq. (3) is negligible, is that the value of the term contributed less than one percent of the total on the left side of the equation. If the terms for $n > 1$ are neglected, (3) can be rewritten

$$wk/Hl = \alpha t/L^2 = 1/6 + 2/\pi^2 e^{-(\pi/L)^2 \alpha t}$$

When the value at $\alpha t/L^2 = 0.4400$ is substituted

$$wk/Hl = 0.2759 = 0.4400 - 0.1667 + 0.0026$$

Since $0.0026 < (0.01)$ (0.2759) the condition has been met.

If the $n = 2$ term had been included, its value would have been $(2/4\pi^2)e^{-4\pi^2/L^2 \alpha t} = 0.0508 e^{-17.55}$

which is clearly negligible.

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