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An Equation for the Entrainment of Ejectors

HAROLD J. HOGE* AND RONALD A. SEGARS†
U. S. Army Natick Laboratories, Natick, Mass.

IN a recent paper[1] the condition of generalized choking for two frictionless, one-dimensional streams flowing side by side in pressure equilibrium was shown to be

$$\frac{k'+1}{2k'}\left(1-\frac{1}{\lambda'^2}\right)A' + \frac{k''+1}{2k''}\left(1-\frac{1}{\lambda''^2}\right)A'' = 0 \quad (1)$$

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* Head, Thermodynamics Laboratory, Pioneering Research Division.

† Member, Thermodynamics Laboratory, Pioneering Research Division.

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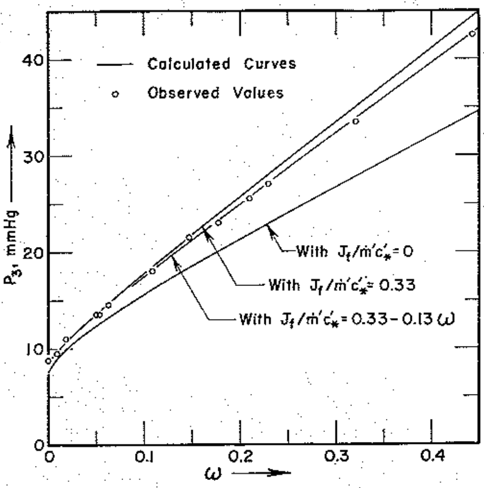

Fig. 1 Observed and calculated initial pressures of the driven stream as a function of entrainment ratio, for Run 168.

Here k is the isentropic exponent, A is the cross-section area of a stream, and $\lambda \equiv u/c_*$ is its velocity number; u is the actual velocity of a stream and c_* is the sonic velocity that the stream would reach at the throat of a converging-diverging nozzle. Primes indicate the driving stream and double primes the driven stream. This equation is obtained by minimizing $J' + J''$, the sum of the impulse functions of the two streams.

In Ref. 1 an expression for the pressure-area product of a stream is given from which it can be shown that

$$\frac{PA''}{PA'} = \frac{\dot{m}''}{\dot{m}'}\frac{c_*''}{c_*'}\frac{[(k''+1)/2k''][\lambda''+(1/\lambda'')]-\lambda''}{[(k'+1)/2k'][\lambda'+(1/\lambda')]-\lambda'} \tag{2}$$

where $\dot{m}$ is the mass rate of flow of a stream and $\dot{m}''/\dot{m}'$ is of course the entrainment ratio ω. We can eliminate A''/A' between Eqs. (1) and (2) to obtain (note that P drops out)

$$\omega = \frac{c_*'}{c_*''}\,\frac{\dfrac{k'+1}{2k'}\left(1-\dfrac{1}{\lambda'^2}\right)\dfrac{k'+1}{2k'}\left(\lambda'+\dfrac{1}{\lambda'}\right)-\lambda'}{\dfrac{k''+1}{2k''}\left(1-\dfrac{1}{\lambda''^2}\right)\dfrac{k''+1}{2k''}\left(\lambda''+\dfrac{1}{\lambda''}\right)-\lambda''} \tag{3}$$

This equation gives the entrainment ratio as a function of the velocity numbers of the two streams; it embodies the conditions of minimum total J and equality of pressures of the two streams, and it holds at the plane of generalized choking. This equation gives a means of calculating entrainment in situations in which generalized choking occurs; generalized choking is most likely to occur when the entrainment is not too small and (probably) when the mixing tube is not too long. Also, like ordinary choking, generalized choking will occur only if the downstream pressure is below a certain maximum value.

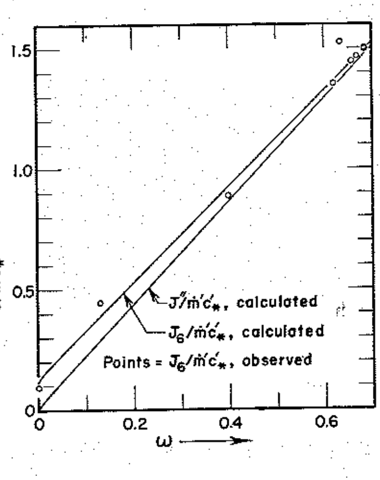

Fig. 2 Observed and calculated values of the impulse function, data of Eastman.[2]

To use the entrainment equation (3) we must first make some assumptions about λ'; we can then calculate ω as a function of λ''. In addition we must relate ω through λ'' to the initial pressure of the driven stream since the object of a calculation of entrainment is to predict ω as a function of this initial pressure. Our experimental data led us to the conclusion that in many ejectors (jet compressors) λ' does not change much as ω changes, and we have investigated Eq. (3) by setting λ' = a constant; usually this constant is the value λ_5 that describes the stream as it leaves the driving-gas nozzle. The assumption of constant λ' is better than it looks at first sight, for λ' is likely to lie between 1 and 2, and in this range the numerator of Eq. (3) is a slowly-varying function. Hence a change in λ' of say 10% would change the calculated ω by much less.

For a constant-area mixing tube, the conditions in the plane of choking can be related to conditions at the mixing-tube entrance by equating the impulse functions of the two planes. The impulse function is $J = PA + \dot{m}u$; it can also be written in the form[1]

$$J = \dot{m}c_*[(k+1)/2k][\lambda + (1/\lambda)] \tag{4}$$

Let J_5 and J_6 be the impulse functions of the driving and driven streams, respectively, as the streams enter the mixing tube, and let J_f be the loss in J due to wall friction, between the entrance of the mixing tube and the plane of choking. Since the mixing tube has a constant area, we must have

$$J_5 + J_6 = J' + J'' + J_f \tag{5}$$

The simplest possible assumption that will permit us to go further is to set $J_f = 0$. Also, since we assumed $\lambda' = \lambda_5$ we have $J' = J_5$, and hence Eq. (5) reduces to $J'' = J_6$; this requires $\lambda'' = \lambda_6$ and there is no change in conditions between the mixing-tube entrance and the plane of generalized choking.

A better assumption than to ignore friction is to assume a constant value for J_f; then $J_6 = J''$ + a constant, and λ_6 can be found from J_6. Still another approach is to assume that J_f is a linear function of ω. Once λ_6 has been found, all the conditions in state 6 can be calculated from ω, A_6, and the stagnation properties of the stream. Finally, we may assume that upstream of state 6 the driven stream is isentropic. This permits its initial pressure P_3 to be calculated; we now have ω as a function of P_3, and the goal of the calculation has been reached.

Figure 1 shows a set of our own data (Run 168) for air entrained by Freon-12. The plotted points are the observed values of P_3, shown as a function of ω. If P_3 is calculated from Eqs. (3) and (5), assuming $J_5 = J'$ and $J_f = 0$, the lowest curve in Fig. 1 is obtained. Although this calculation ignores friction altogether, it shows the proper form of the relation between P_3 and ω. In particular, it shows that there is no appreciable entrainment until P_3 reaches 7- or 8-mm Hg, in fair agreement with the experimental data. The value of P_3 when $\omega = 0$, sometimes referred to as the "base pressure," is predicted with fair accuracy. Better agreement with the data is obtained by assuming a constant nonzero friction loss; if we put $J_f/\dot{m}'c_*' = 0.33$, and otherwise calculate as before, we obtain the top curve in Fig. 1. The value 0.33 was chosen to give good agreement with the data at low values of ω; at large values of ω the curve is a little too high. Still better agreement with the data is obtained if we let J_f vary with ω. Adopting the semiempirical equation $J_f/\dot{m}'c_*' = 0.33 - 0.13\,\omega$, the middle curve in Fig. 1 may be calculated. It represents the experimental data almost perfectly. The values of J_f used in the preceding calculations are not necessarily those that would be calculated from conventional friction coefficients. They are of the same order of magnitude, but we are using J_f as a catchall to account for friction and also to account for inaccuracies in our approximations; for example, if J' varies slightly, so that our assumption that $J' = J_5$ is not quite correct, we may be compensating for this in our choice of J_f.

Another set of data that also shows good agreement with the present theory is shown in Fig. 2; they are for air driven by CO_2 and are taken from Eastman.[2] Here the impulse function has been plotted rather than the pressure P_3. The plotted points are the observed values of $J_6/\dot{m}'c_*'$ and the lower curve is the calculated value of $J''/\dot{m}'c_*'$. If there were no friction we should expect to find $J'' = J_6$, and this curve would represent the data. A better fit is obtained by putting $J_f/\dot{m}'c_*' = 0.115 - 0.14\ \omega$; a calculation employing this value of friction gives the upper curve in the figure.

It may be seen in Fig. 2 that the experimental data show J_6 to be very nearly a linear function of ω. This linear relation we have noted in many sets of data; it was observed as an empirical fact before the present method of calculating entrainment was developed, and it appears to be quite general. The present theory of entrainment predicts this linear relation between J_6 and ω. As may be seen in Fig. 2, J'' is very nearly a linear function of ω. One would hardly guess this from an inspection of Eqs. (3) and (4) from which ω and J'' are calculated; we have tried to put the equations into a form that would make the linear relation obvious, so far without success. Adding a linear friction allowance to J'' gives J_6 as a linear function of ω, a sort of Hooke's law for ejectors.

The methods of the present note were used to calculate a set of data for an air:air system taken from Fig. 7 of a paper by Fabri and Siestrunck[3]; the data labeled 5.5 were used. From the nozzle dimensions we calculate that λ_5 should equal 1.78, and this value was used for λ'. With $J_f/\dot{m}'c_*' = 0.052$, a calculated curve was obtained that agreed fairly well with the observations; the agreement was not as good as Fabri and Siestrunck obtained, but the curvature at low ω's was better matched. A good fit of the data could be obtained by increasing the numerator of Eq. (3) and using a much larger value of friction, but these changes could be justified only on empirical grounds.

In closing we will point out that the use of Eq. (3) plus the assumption of constant λ' makes it possible to derive similarity laws for entrainment. If the behavior of a steam-jet ejector in entraining air is known and we wish to know how the ejector will behave when CO_2 is entrained, we have only to replace the denominator of Eq. (3) with a new denominator corresponding to CO_2, and take into account the small difference in pressure drop required to produce the same λ'' in both air and CO_2. Part of the difference in behavior of different gases is expressed in the different values of k, but most of the difference is in the values of c_*. Since c_* is inversely proportional to the square root of the molecular weight of the gas, we may expect ω to be directly proportional to the square root of the molecular weight of the gas being entrained, when variations in k are neglected. The curve experimentally obtained by Holton[4] and accepted as standard by the Heat Exchange Institute is, in fact, roughly of the form thus predicted.

References

[1] Hoge, H. J. and Segars, R. A., "Choked flow: a generalization of the concept and some experimental data," AIAA J. **3**, 2177–2183 (1965).

[2] Eastman, R. H., "Effects of a particle-laden driving stream on ejector efficiency," Joseph Kaye and Co. Rept. 52, AD-411486 (March 1963).

[3] Fabri, J. and Siestrunck, R., "Supersonic air ejectors," Advances Appl. Mech. **5**, 1–34 (1958).

[4] Holton, W. C., "Effect of molecular weight of entrained fluid on the performance of steam-jet ejectors," Trans. Am. Soc. Mech. Engrs. **73**, 905–10 (1951).