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ECONOMIC APPLICATIONS OF SWEETNESS SCALES

INTRODUCTION

THE PRESENT STUDY concerns the economics of sweetness mixtures, in which pairs of sweeteners are used in conjunction in order to increase the overall sweetness of a product. The different costs of materials such as sucrose, glucose (dextrose), saccharin and cyclamate provide possibilities for considering mixtures in terms of a model that minimizes overall cost while maintaining sweetness, or maximizing sweetness while maintaining cost.

The tastes of sweetener mixtures have been studied by food scientists and psychologists for the past five decades. Early work by the sugar industry focused on the possibility that inverting sugar (sucrose) to a mixture of glucose and fructose would enhance sweetness, and thus enzymatic changes of sucrose could provide greater sweetness at the same cost (Sale and Skinner, 1922). At about the same time in Germany a number of studies on the taste of mixtures of the artificial sweeteners, saccharin and dulcin (Paul, 1922) were underway as a result of a shortness of sucrose. The results of these experiments indicated that mixtures were significantly sweeter than their components alone.

The method for determining how sweetnesses 'add' together in a mixture has been elucidated by Cameron (1943; 1944; 1945; 1947). Cameron asked his subjects to taste two solutions of different sugars and then to select the concentration of sucrose that matched the sweetness of each sample (sucrose equivalent). He subsequently mixed together the sugars and repeated the experiment. The results were expressed as three 'sucrose equivalents,' two for the unmixed components and one for the mixture. Additivity of sweetness occurred when the 'equivalent' for the mixture equalled the arithmetic sum of the unmixed 'equivalents.' In a large series of experiments Cameron demonstrated that additivity occurred for pairs of sugars, but only when glucose, maltose or lactose was used as the reference sugar. Expressing sweetness in sucrose or fructose equivalents did not result in additivity of sweetness.

These early studies by Paul and Cameron lacked a true measure of subjective sweetness, and relied on the concentration of glucose or sucrose as the implicit sweetness unit. Recent work by Stevens (1953; 1960; 1969) demonstrated that subjects may be directed to give numerical judgments in proportion to subjective magnitude, taste intensity included, and that these numerical estimates, called 'magnitude estimates,' provided meaningful ratio measures of taste intensity. For example, a sweetness judgment of 20 means twice the sweetness of a judgment of 10, and eight times the sweetness of a judgment of 2.5. Moskowitz (1970a; 1970b; 1971a) reported a series of sweetness scales for several dozen different sugars and several artificial sweeteners. Details of the experimental technique are provided by Moskowitz (1970a).

A convenient and systematic outcome of these direct scaling studies was the finding that the numerical judgments of sweetness could be related to molar or percentage concentration by the simple power function $S = kI^n$. That is, S represents the sweetness judgment and I represents molarity. The exponent n and the intercept k may be obtained from the straight line that results when the power function is plotted in log-log coordinates, to yield the equation $\log S = n \log I + \log k$. The slope of the line provides the exponent, and the intercept provides the value for log k.

The exponent n is the critical parameter for the sweetness equation because it governs the rate at which sweetness increases with concentration. It appears to exceed 1.0 for sugars (Stevens, 1969; Moskowitz, 1970a; 1970b; 1971a), but is less than 1.0 for saccharin and cyclamate. When n exceeds 1.0 sensory magnitude accelerates or grows more rapidly than molar concentration, whereas for n less than 1.0 the opposite occurs and sensory magnitude grows less rapidly. Very low values of n would indicate that large increments of concentration scarcely produce any changes in perceived sweetness. The intercept, or multiplier, k, depends upon the size of numbers selected by the subject and upon the measure of concentration selected. However, when several sugars are rated for sweetness in the same session and their exponents n are made equal either experimentally or by subsequent statistical analysis, then k reflects the ratio of sweetness among different sweeteners (Moskowitz, 1970a). This is because k reflects the relative distance in logarithmic values (i.e., ratio) of two parallel lines.

Recent work in sweetness has attempted to combine power functions of sweetness in order to predict mixture sweetness. Papers by Stone and Oliver (1969), Stone et al. (1969) and Yamaguchi et al. (1970) have tried various combinations of sweetness functions. Usually, however, some multiplicative constant is needed to account for the often-observed result that there is 'synergism,' so that the sweetness of the mixture exceeds the predicted sweetness.

Because of the synergistic effect in mixtures, a combination of power functions and an associated multiplier to handle the effect is shown below:

Sweetness $S_a = k_1\ C_a^m$; Sweetness $S_b = k_2\ C_b^n$

Mixture sweetness $S_{a,b} = k_3\ (k_1\ C_a^m + k_2\ C_b^n)$

Empirical studies of mixtures (Stone and Oliver, 1969; Stone et al., 1969; Moskowitz, 1971b) suggest that the values of k_3 for synergistic mixtures are moderately greater than 1.0, e.g., about 1.4–1.8, so that the actual prediction made by summing simple power functions is an underestimate. For convenience in simulation we may assume that k_3 remains unchanged across most of the sweetness range and may be viewed simply as a 'change-of-scale.' The form of sweetness summation is unaffected if k_3 is permitted to vary to correct the under- or overpredictions.

EXPERIMENTAL

Procedure

In three experiments glucose was evaluated for sweetness in conjunction with fructose, sodium cyclamate and sodium saccharin. In each experiment subjects received seven solutions of glucose, seven solutions of the second sweetener, and 34 mixtures of the two sweeteners in varying ratios and dilutions. Samples were served to the subjects in small, ¾ oz paper cups and maintained at the room temperature (22°C). The solutions were made up three days prior to the experiment and stored under refrigeration, to permit both mutarotation to an equilibrium mixture of isomers and to prevent mold growth. S's were instructed to judge only the sweetness of the solutions, and for the simple unmixed sweeteners power functions of the form $S = kI^n$ were fitted to the median judgments of 24 S's. Because of experi-

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mental variation the exponents and intercepts of the glucose function varied across experiments (as shown later by the 'generating functions' in the figures), although in each instance

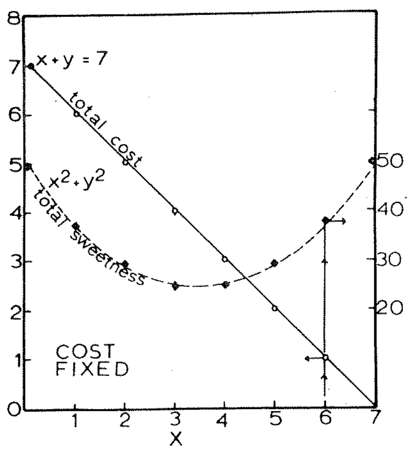

Fig. 1—A hypothetical iso-cost function in which X and Y are two sweeteners, each having unit cost. Total cost = \$7. Sweetness is assumed to be equal to the square of molar concentration (total sweetness = $X^2 + Y^2$). All points on the solid line satisfy the equation $X + Y = 7$. All points on the dashed curve satisfy the function $F(X,Y) = X^2 + Y^2$, or $G(X) = X^2 + (7-Y)^2$. To obtain a value for Y for any X, use the solid line. The total 'sweetness' is given by the value of the right-hand ordinate, labelled $X^2 + Y^2$.

the glucose exponent was higher than 1.0 (about 1.3–1.6). Power functions also described the sweetness of fructose. Power functions were forced to fit the saccharin and cyclamate functions, even though they demonstrated significant nonlinearity in log-log coordinates. A revision of the simple summation model may be made to account for quadratic and cubic terms in the saccharin and cyclamate functions. For ease of computation, however, only their linear portions (i.e., simple power functions) were used.

For each sweetener the cost of the mixture was ascertained from the prices of the ingredients. The cost per mole was obtained from the 1971 catalog of the Sigma Chemical Co. and reflects the cost of reagent-grade material. For each mixture of two ingredients, therefore, there are two associated values: a total cost obtained from a simple linear sum of independent costs and a total sweetness based upon the addition of two power functions.

Other pairs of sweeteners were also investigated, but only by computer simulation. For these 'hypothetical' mixtures the individual power functions relating sweetness to concentration were obtained from Moskowitz (1971b). For both the empirical and the hypothetical mixtures the value of k_3, which accounts for synergistic effects was arbitrarily set at 1.0 to facilitate computation and to permit comparison of the various mixtures with each other.

Types of simulation

Two theoretical problems were considered: maximization of sweetness subject to maintaining a fixed cost and minimization of total cost subject to maintaining constant sweetness. Initial approaches were to solve analytically these problems with the help of Lagrange multipliers (Taylor, 1955) and with the appropriate cost and sweetness functions. In all cases where the exponents were not small whole numbers the analytic solution did not work, and thus a computer simulation was needed.

In the computer simulation the overall cost of the mixture was first fixed, and a large number of pairs of concentrations were then computed that satisfied the cost constraint. A smooth curve was drawn showing the relative amounts of the two sweeteners whose overall cost was the desired amount. For each pair the sweetness was then calculated. Then a curve was drawn showing the sweetness of the mixture at the different levels of sweeteners for the fixed total cost. With this method several different overall costs were scanned in order to produce different cost functions (iso-costs), and their corresponding contours were computed. In the second part of the simulation the overall sweetness was fixed and a large number of pairs of concentrations were determined by solving the sweetness equation. The total cost of each mixture was then calculated. Again, several different levels of total sweetness were scanned in order to determine the sweetness contours, and the costs of these mixtures were subsequently calculated.

RESULTS

FIGURE 1 illustrates a straight line (total-cost contour) for a hypothetical pair of sweeteners X and Y. The 'cost' of 1 mole of X and of Y is 1 unit respectively for each. For each value of X there is

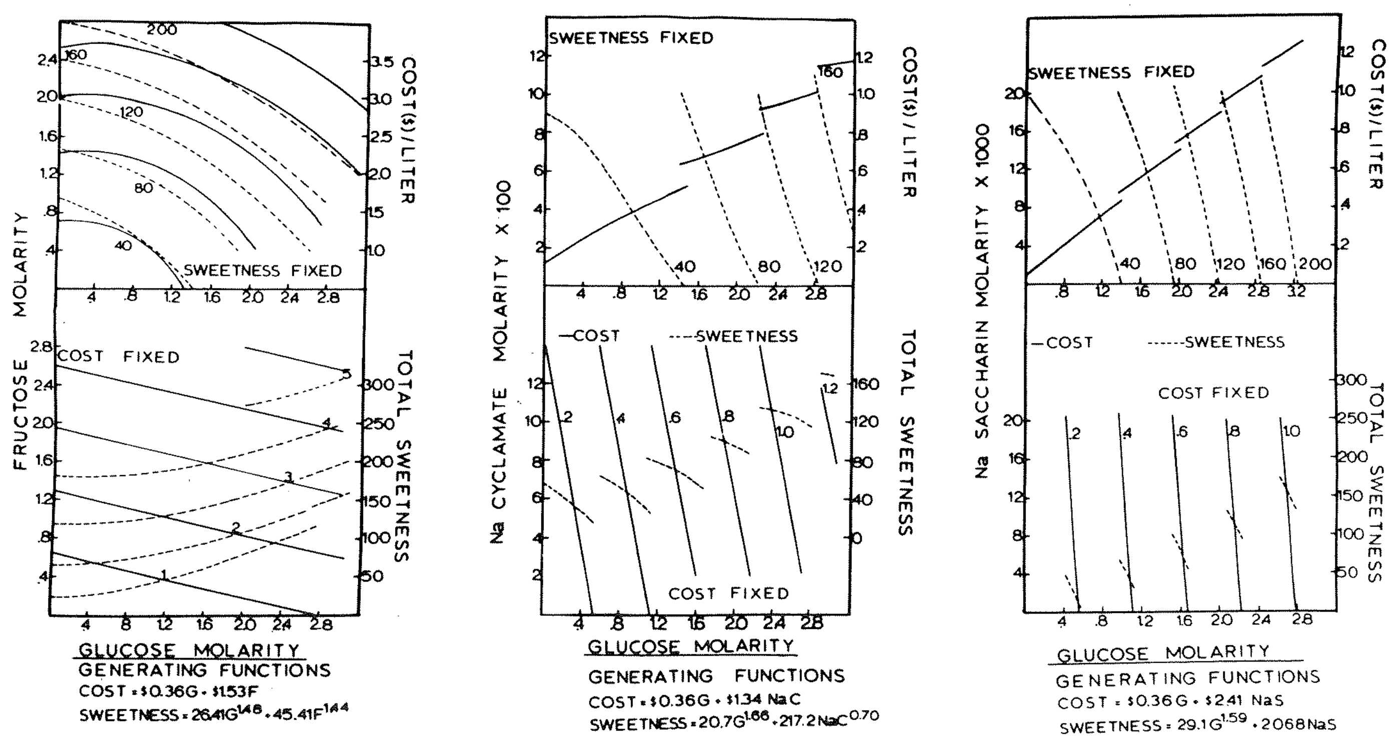

Fig. 2—Mixtures between glucose and fructose, Na cyclamate and Na saccharin. The generating equations are shown below each pair of graphs. Numbers on the top figure in each pair indicate the overall, fixed sweetness level, whereas those in the lower portion indicate overall, fixed cost. All pairs in this figure were investigated in actual experiments, although the curves are idealized versions of the empirical mixture data. Cost is given in \$ per mole weight of the mixture.

only one value of Y that satisfies the cost constraint $X + Y = 7$. Only positive values of X and Y are shown, since negative values indicate that a concentration must be subtracted from the mixture. Sweetness is assumed in this example to be represented by the square of molar concentration (so that a 4:1 increase in molarity would lead to an increase of 16:1 in sweetness). In addition, the sweetnesses of the components X and Y are assumed to add algebraically. Thus, the curve represented by $X^2 + Y^2$ represents the overall sweetness of the mixture, and its numerical value may be obtained from the vertical axis at the right of Figure 1.

For any value of X, the corresponding value of Y can be found that satisfies the constraint, and the overall sweetness of the mixture can be calculated. In fact, both variables, Y and $(X^2 + Y^2)$, are uniquely determined for any value of X. The sweetness curve may thus be considered either as a function of both X and Y (i.e., $X^2 + Y^2$) or as a function of X alone [i.e., $X^2 + (7-X)^2$]. This unique determination of the sweetness function arises from the cost constraint, which makes Y directly depend upon X. A similar figure may be constructed for the dual problem, of computing overall cost when sweetness is maintained at a constant value (e.g., $X^2 + Y^2 = 10$). Total cost in the dual problem would be given by the equation $X + (10-X^2)^{.5}$.

Figure 2 shows the mixture functions obtained from three empirical studies. Below each part of the figure are the sensory functions that were used to generate sweetness values, as well as the cost functions used to compute the cost per mole of hybrid mixture. The overall sweetness of the mixtures was fixed at five different values: 40, 80, 120, 160 and 200. In the present system the sweetening power of 0.5M glucose (9%) has been assigned a sweetness value of 10. Because of experimental variations, the sweetness functions for glucose, as shown in the bottom equations of Figure 2, differ slightly among themselves so that the exponent varies between 1.3 and 1.6.

For each of the three mixture-sets in Figure 2, the horizontal axis represents the molarity of glucose. The computer analysis scanned a large number of concentrations between 0.0 and 3.0 moles. The corresponding molarity of the second sweetener satisfying either the cost or the sweetness constraint is shown at the left hand side of the vertical axis.

In order to use the figures, one must first locate the contour that is of interest. For example, consider the mixture whose overall sweetness is 40. A large number of glucose concentrations satisfy this requirement, and for each concentration a value for fructose may be found. One possible pair is 0.4 moles of glucose and 0.8 moles of fructose (approximately). The overall cost of these two sweeteners may be obtained by first extending a vertical line upwards from the horizontal axis (at 0.4 molar glucose). The cost contour (solid curve) intercepts this vertical straight line at a value given at the right hand side of the figure. The analogous reasoning is applicable to situations in which the total cost of the mixture is fixed at a single level and sweetness is to be found.

Two primary results merit discussion. First, there are different 'feasible ranges' of mixtures that satisfy the constraints of sweetness or cost. For example, mixtures of sugars (e.g., fructose and glucose) provide large ranges of concentrations with which to work, whereas markedly narrower ranges are found when a sugar is mixed with an artificial sweetener of greater potency. Second, there are different 'cost ranges.' Sugar mixtures are significantly more expensive than mixtures of sugar with artificial sweeteners (in some cases 4x more expensive). Therefore, when sweetness of the product is the major consideration, artificial sweeteners provide reduced costs.

When the overall cost of the mixture is maintained at a fixed value (the opposite problem to maintaining overall sweet-

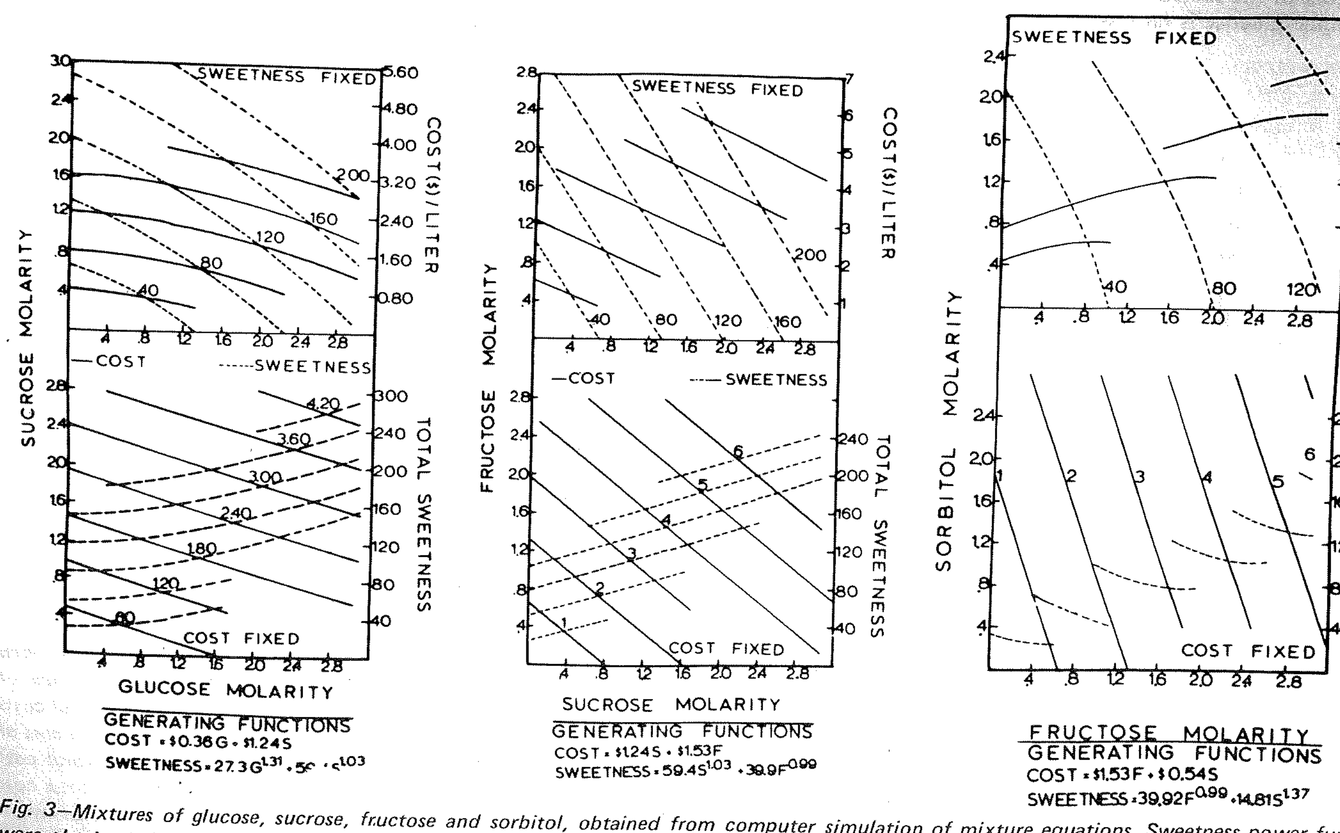

Fig. 3—Mixtures of glucose, sucrose, fructose and sorbitol, obtained from computer simulation of mixture equations. Sweetness power functions were obtained from Moskowitz (1971a).

ness), the mixtures again show different behaviors. For fixed total cost, glucose-fructose mixtures become sweeter with increasing amounts of glucose, a result that is due to the smaller cost of glucose. That lower cost of glucose is sufficient to outweigh the greater sweetening power of fructose. On the other hand, mixtures of glucose with cyclamate or saccharin become less sweet (for fixed cost) as glucose predominates in the mixture. This latter result is obtained because a great deal of sweetness can be obtained for relatively small amounts of the artificial sweetener.

Figure 3 presents the results of computer simulation of mixtures between glucose, sucrose, fructose and sorbitol. These four sugars are commonly used by the food industry to provide sweetness and represent a relatively moderate variation of costs and relative sweetness.

The sweet functions were obtained from estimates provided by Moskowitz (1971a). Sucrose and fructose are the sweetest of the two sugars and grow more slowly in sweetness than either glucose or sorbitol (Cameron, 1947; Moskowitz, 1970a; 1971b). The sweetness curves of glucose and sorbitol are parallel in log-log coordinates, as are the curves for sucrose and fructose. Below each set of functions are the generating equations that were used in the simulation.

Glucose-sucrose and fructose-sucrose mixtures follow similar contours. For example, at a fixed sweetness when the concentration of glucose is increased, the overall cost of the mixture decreases. This occurs since glucose is far less costly than sucrose. For fixed costs the cheapness of glucose far outweighs the sweetness advantage of sucrose. Similar arguments may be made for mixtures of fructose and sucrose wherein fructose is more expensive than sucrose.

When fructose is combined with sorbitol, however, the factors of cost and of sweetness compete against each other. Sorbitol is less expensive than fructose but fructose is much sweeter. The result is a slight increase in the sweetness of a mixture with increases in fructose content when the cost is held constant. As a general rule then, mixtures of this type in which the cost favors one material and the sweetness favors another, material will tend to have flatter contours. This is especially true if costs and sweetness ratios are approximately commensurate and counterbalance each other. Steep contours will occur when one dimension (cost or sweetness) markedly overrides the other.

Figure 4 shows a series of sorbitol-glucose mixtures in which the sweetness was fixed at one of four values (40, 80, 120, 160). For each of several prices of sorbitol (e.g., \$0.10 per mole) the cost function was traced out. The result is a series of cost contours for each sweetness level. The shape of the contour changes as the price of sorbitol is systematically increased. At low sorbitol costs (e.g., \$0.10 per mole) increases in glucose, the more expensive sugar raises the cost of the mixture. At intermediate sorbitol costs (e.g., \$0.30 per mole) there are two optimum points, either very high concentrations of sorbitol or very high concentrations of glucose. Finally, with high sorbitol costs (e.g., \$0.50), the price of sorbitol militates against using it, and the best strategy is to use only glucose. This approach to tracing out the several possible contours elucidates the type of strategy that might be used when a single ingredient systematically varies in cost, but can be replaced by another material possessing many of the same properties.

Finally, Figure 5 illustrates the contours that are obtained when cost is maintained at a given level (either \$0.40, \$0.80 or \$1.20 per liter of sweetener mixture) and the price of sorbitol systematically varies. The order of the cost functions in the figure is maintained for the sweetness function, so that the uppermost cost function corresponds to the uppermost sweetness function. Large changes in sorbitol, when it is inexpen-

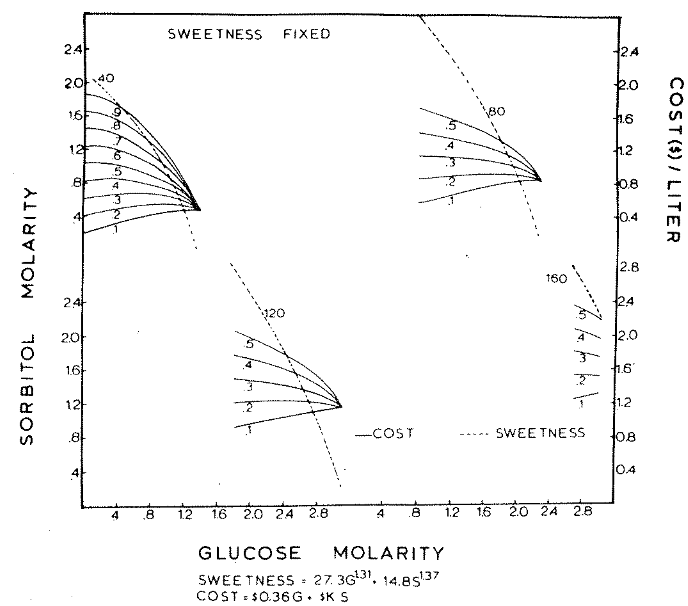

Fig. 4—Contours for glucose and sorbitol when overall sweetness of the mixture was fixed at four values. Each of the solid lines reflects the overall cost of the mixture when the cost of sorbitol is varied at 10:1 range (from \$0.10–1.00 per mole).

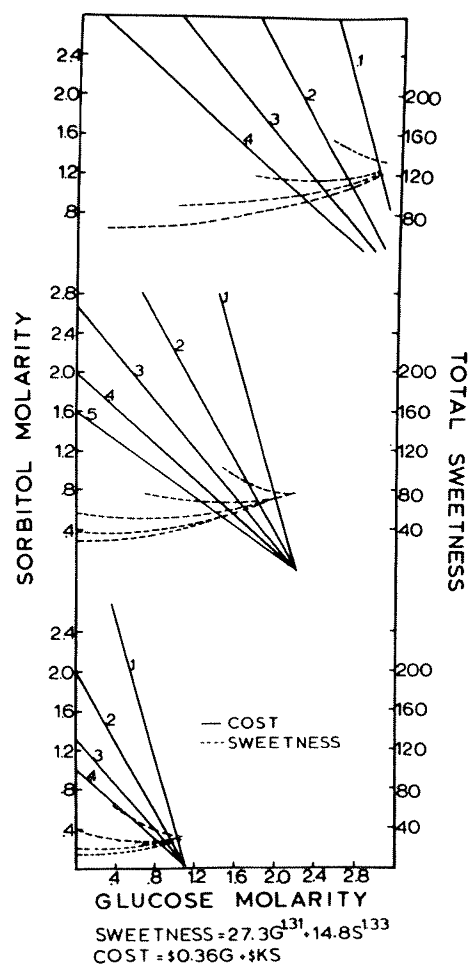

Fig. 5—Contours for glucose and sorbitol when the overall cost of the mixture is fixed at one of three values. For each total cost, sorbitol costs have been varied over a 5:1 range. Each cost of sorbitol yields a new contour for total cost (solid line) and for each contour of total cost there is a comparable contour for total sweetness.

sive, can be tolerated and offset by small changes in glucose without affecting the cost function. The optimum then is to use all sorbitol, with concomitant increases in sweetness. Intermediate prices of sorbitol for fixed total cost reduce the high sweetness when large amounts of sorbitol are used (since the prices are commensurate for the two sugars but glucose is sweeter). For high costs of sorbitol, both the price and the low sweetness militate against producing a high degree of sweetness when much sorbitol is used, and mixtures tend to have less sweetness with more sorbitol.

DISCUSSION

THE PRESENT STUDY concerns a model system in which the sweetener is sampled in aqueous solution. Similar data should be generated by experimental means to test the approach in specific food products. The technique of magnitude estimation permits this approach to be used with relative rapidity and little expense, and may be applied in actual product development. In addition, the fact that a multiplicative correction must be used to account for synergistic effects in mixtures is not a serious detriment, for it requires only a change-of-scale for sweetness. The values corresponding to psychological sweetness may be multiplied by a correcting factor, so that they represent the actual sweetness levels relative to a standard, or a computer program can be written to account for the multiplier. Many other factors besides overall sweetness enter into the selection of an appropriate sweetener and concentration for a given food. Mixtures that maintain sweetness at a fixed level may not be equally acceptable to the consumer, and specific parameters of each food have to be considered before selecting any one mixture. Hence, the present study provides only two constraints for mixtures: the levels of sweetener that sum to a given cost and to a given sweetness. Other constraints may be the acceptability to the consumer, weight of sweetener and perhaps even caloric value.

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