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Strain-Wave Reflections During Ballistic Impact of Fabric Panels

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ABSTRACT

A numerical model is presented which describes the strain level build-up in yarns due to multiple strain wave reflections from yarn crossover intersections in a woven fabric subject to ballistic impact. Crossing yarns present barriers from which strain waves are partially reflected. The maximum yarn strain occurs at the point of impact and decays with distance along the yarn away from this point. The rapidity of decay is governed by the crossover reflection coefficient. Using observations of the deformation cone size of ballistically impacted fabric panels, it is concluded that the reflection coefficient is small (approximately 0.01). The strain increases with time at different rates for different reflection coefficients until failure at the impact point. Extensions of this model to other fibrous structures are discussed.

KEYWORDS

Yarns; woven fabric; fabric panel. Numerical model; yarn strain; strain level buildup; strain wave reflections; reflection coefficients. Ballistic impact; impact point; decay distance.

Introduction

Textile materials in the forms of fabrics, felts, and webbings are widely used in energy absorbing systems such as automobile seat belts, parachute lines, and body armor. The particular fiber assembly utilized depends on the performance requirements of the application. The problem discussed in this paper arose during an investigation of nonwoven ballistic body armor materials and assemblies for the U. S. Army Natick Laboratories, Natick, Massachusetts. However, the physical phenomenon of strain-wave reflection takes place in many dynamically loaded elastic systems as described below.

When a fibrous assembly is subjected to high speed loads, ballistic impact loading in the case of body armor materials, strain waves are initiated, and propagate away from the point of impact. The exact description of elastic wave propagation in a complex assembly is impossible, practically speaking. Thus, simplifying assumptions must be made about wave speeds, wave amplitudes, and structural interactions.

The basic property of elastic wave propagation of importance here is the fact that the strain level in a yarn is doubled by a wave reflected from a fixed end [4]. From a free end, the algebraic sign of the wave amplitude is reversed, so that the waves cancel and the strain level is zero. The ideal fixed and free ends are extremes. In a fiber assembly, yarn or fiber crossovers behave as partially fixed ends from which an incident strain wave is partly reflected and partly transmitted past the crossover. Thus, in a fabric, the crossovers magnify the strain level compared with an idealized free yarn system. Depending on the magnitude of the reflection coefficient, higher strain levels can lead to decreased performance of body armor systems.

Description of Wave Amplitudes

Let us consider an impacted straight filament crossed by uniformly spaced filaments. In addition to the easily recognized transverse wave defined by the characteristic "tent" formed by the displaced filament [1], a longitudinal strain wave travels along the filament. Wave velocity dispersion effects are neglected, and it is assumed herein that the longitudinal strain wave speed c in the filament is given by the infinite medium plane wave speed $(E/\rho)^{\frac{1}{2}}$, where E is Young's modulus and ρ the filament density; thus, all creep and stress relaxation effects are also neglected.

The crossing filaments are taken to be partially reflecting barriers to the passage of the longitudinal wave. A reflection coefficient K $(0 \leq K \leq 1)$ is used to describe the fraction of the impinging longitudinal

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wave amplitude reflected by the crossover. A transmission coefficient P describes the fraction of the wave transmitted past the crossover. Since the used model assumes no losses $K+P=1$. All crossovers are further assumed to be governed by the same K and P. The midpoint of the filament, that is, the impact point, is assumed to be perfectly reflecting. All strain waves coming in contact with this boundary are reversed in direction of travel with their amplitudes unchanged.

Since the problem is symmetric about the impact point ($J=1$), only the right half will be considered. Upon impact, a longitudinal wave of amplitude A propagates down the filament. Arriving at the first filament crossover ($J=2$), a wave of amplitude KA is reflected and a wave of amplitude PA is transmitted. The transmitted wave arrives at the second crossover $J=3$ to be partially reflected KPA and partially transmitted P^2A. The waves up to $J=5$ for the first five time increments I are shown in Figure 1. The superposed arrows denote direction of travel.

In general, the amplitude $A\ (I, J)$ of the strain wave for any $I=1$ to N and $J=1$ to N is given by the following expressions.

Time	Position	Amplitude
I	J	$A\,(I, J)$
Even	Odd	$A\,(I-1, J)\times(-K)+A\,(I-1, J+1)\times P$
		$J=1$ to $N-2$
	Even	$A\,(I-1, J-1)\times(P)+A\,(I-1, J)\times(-K)$
Odd	Odd	$A\,(I-1, J-1)\times(P)+A\,(I-1, J)\times(-K)$
		$J=2$ to $N-2$
	Even	$A\,(I-1, J)\times(-K)+A\,(I-1, J+1)\times(P)$
	1	$A\,(I-1, 1)\times(-1)$
Any	$N-1$	$A\,(N-1, N-1)\times(-K)$
	N	$A\,(N-1, N-1)\times(P)$

(1)

A plus sign denotes a wave traveling from left to right and the negative sign from right to left. The artificial initial condition $A\,(0,1)=-A$ is required to start the numerical procedure.

The total strain amplitude at any position J along the filament length at any time $I=N$ is given by the cumulative sum of the absolute values of all the strains given in Figure 1 at the position J for all time increments from $I=1$ to N.

Numerical Summation of Wave Amplitudes

The above equations were programmed for a digital computer by NLABS Data Analysis Office and the expressions were evaluated for a series of values of $K (0 \leq K \leq 1)$. The changes in amplitudes of the strain at each position J and time I were determined as well as the cumulative total strain amplitudes.

I \ J	1	2	3	4	5
1	$\overrightarrow{A}$				
2	$\overleftarrow{KA}$	$\overrightarrow{PA}$			
3	$\overrightarrow{KA}$	$\overleftarrow{KPA}$	$\overrightarrow{P^2A}$		
4	$\overleftarrow{K^2A}$ $\overleftarrow{KP^2A}$	$\overrightarrow{K^2PA}$ $\overrightarrow{KPA}$	$\overleftarrow{KP^2A}$	$\overrightarrow{P^3A}$	
5	$\overrightarrow{K^2A}$ $\overrightarrow{KP^2A}$	$\overleftarrow{K^3PA}$ $\overleftarrow{K^2PA}$ $\overleftarrow{KP^3A}$	$\overrightarrow{K^2P^2A}$ $\overrightarrow{K^2P^2A}$ $\overrightarrow{KP^2A}$	$\overleftarrow{KP^3A}$	$\overrightarrow{P^4A}$

FIG. 1. Amplitudes of propagating reflected waves.

The results are illustrated in Figures 2–4. Figures 2–3 give the strain wave amplitude decay with distance for increasing time after impact for two values of the reflection coefficient K. As shown, the maximum strain occurs at position 1, the point of impact, and increases with time. An initial strain wave of unit

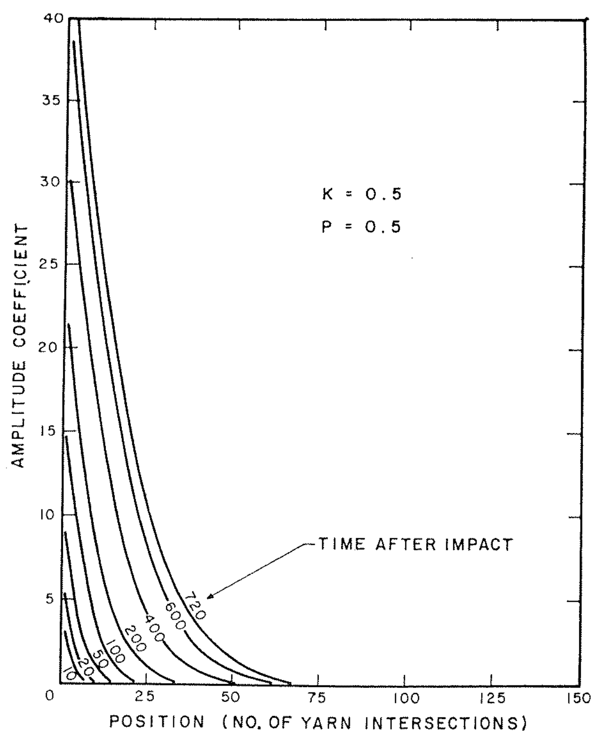

FIG. 2. Amplitude coefficient vs position for successive time increments ($K=0.5$).

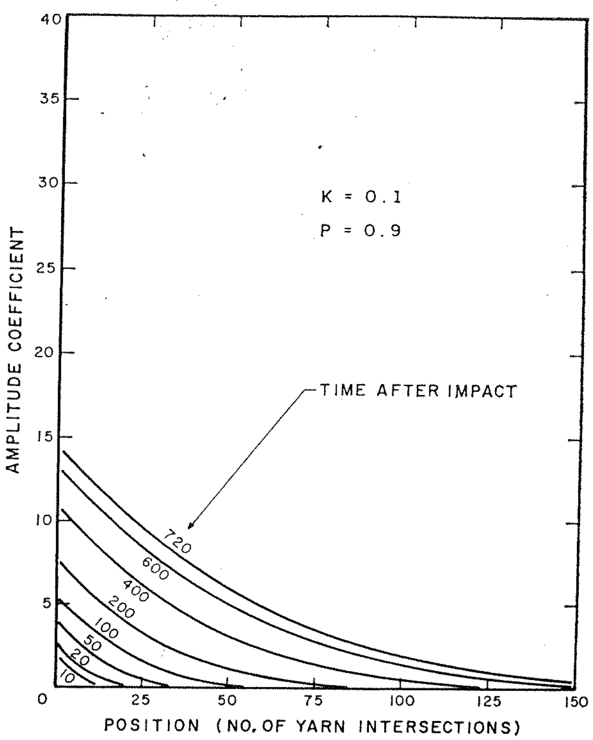

FIG. 3. Amplitude coefficient vs position for successive time increments ($K = 0.1$).

amplitude was selected for convenience, so the ordinate units are multiples of the initial wave amplitude. The time axis units are multiples of the transit time of the wave between crossovers. For example, if the yarn crossover spacing is denoted by N(yarns/ft) and the longitudinal wave speed c(ft/sec), then the time axis unit is $t(\text{sec}) = 1/Nc$. The strain buildup at the impact point, position 1, as characterized by the strain amplitude coefficient is shown in Figure 4 for a series of reflection coefficient K values.

The initial strain amplitude ϵ_0 of the wave generated upon transverse impact by a missile traveling at velocity V, taken from single yarn analysis [3], is

$$\epsilon_0 = \left(\frac{1}{4}\right)^{\frac{1}{3}}\left(\frac{V}{c}\right)^{\frac{4}{3}} \tag{2}$$

Let us consider a yarn with $c = 10{,}000$ fps transversely impacted by a missile at 1000 fps. From Equation 2, under these conditions and initial strain wave with an amplitude of approximately 3% is generated. If the rupture strain of the yarn is assumed to be 21%, then the amplitude coefficient of 7 represents yarn rupture. Diagrams for various K values similar to Figure 2 indicate that the yarn ruptures after 25 time increments for $K = 0.5$, after 50 time increments for $K = 0.3$ and after 175 time increments for $K = 0.1$. The corresponding regions of influence can be read from the abscissas of the diagrams. If the number of crossing yarns is taken to be 30 per in., which is representative of ballistic fabrics, and the region of influence as 3 in. (from visual observation of impacted fabric panels [2]) then $3 \times 30 = 90$ yarn crossovers are within the region of influence. Combining this information with the number of yarn crossovers given in the diagrams for the number of time increments curve that corresponds to a yarn strain amplitude coefficient of 7, it is concluded that for the ballistic body armor structures of current interest $K \cong 0.01$; that is, the yarn crossovers offer little resistance to the wave transmission.

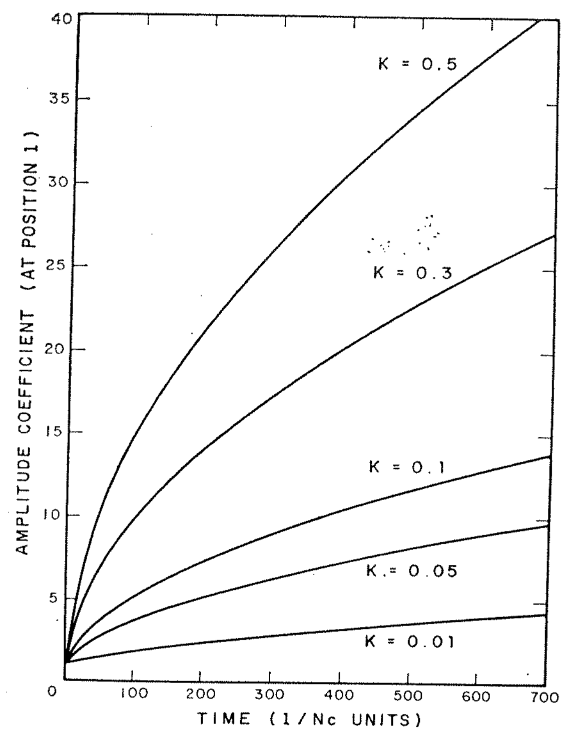

FIG. 4. Strain wave amplitude vs time at impact point.

Discussion

The ever-increasing growth of the stress amplitude at each spatial point is a characteristic of this model. Physically, of course, two additional factors should be recognized. If the fabric panel defeats an impacting missile, the kinetic energy of the missile is reduced to zero which causes the strain amplitude at any spatial point to eventually decrease; this model describes the increasing strain portion of the impact event. On the other hand, if a missile penetrates the panel, the model developed in this paper describes the strain increase to yarn failure.

The model developed above led to the conclusion that yarn crossovers impede very little the propagation of longitudinal waves away from the point of ballistic impact in fabric panels. Any effects of backing material are neglected in the present model. Backing effects, e.g., transverse loads generated by multiple layers, are considered to be worthy of future investigation. Extension of the model to nonwoven materials, such as needle-punched felt, is possible by using a random distribution of crossover points. The expected additional mathematical complexity discouraged such an approach, since those results were not anticipated to be greatly different from the present ones. The curves of amplitude growth at the impact point and the amplitude decay with distance in nonwovens would not be as smooth as for fabrics, but the general conclusion of small impedance would be obtained.

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