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Azimuth Homing in a Planar Uniform Wind

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The planar trajectory of an azimuth homing gliding system maneuvering through a uniform wind field is presented. The kinematics of the motion is discussed generating two first-order differential equations which are separated producing an expression involving only position coordinates. A change of variables is introduced and exact solutions obtained by direct integration. The resulting trajectories fall into two categories classified according to their target-seeking or orbital characteristics. Flight paths in the target-seeking domain are shown to be convergent when the system has a wind penetration capability. The family of target-orbiting solutions are shown to produce trajectories which are captured about the target in elliptic spirals. Launch criteria are established from the time solution which takes the form of an ellipse for all categories of azimuth homing. The definition of a release path as the locus of points from which the flight time necessary to reach the target is constant, follows from this result.

Nomenclature

L/D = lift to drag ratio
t = time
U = horizontal component of total airspeed vector
u = $|U|$, horizontal airspeed
V = system total airspeed vector
α = radial offset angle
W = wind or field velocity
w = $|W|$, wind or field speed
x = horizontal space coordinate fixed to earth
y = horizontal space coordinate perpendicular to x and fixed to earth
z = vertical space coordinate perpendicular to the x-y plane
r = magnitude of the radius vector in polar coordinates
θ = angular position in polar coordinates
λ = u/w wind penetration parameter

Subscript

i = initial

Introduction

AUTOMATIC control of a vehicle in gliding flight poses some interesting problems particularly when aerodynamic performance is limited. A remotely guided recovery system, utilizing some form of gliding deceleration,[1] is an example of such a case. Typically, these systems operate at relatively low airspeed with virtually no capacity for L/D modulation. Discrete or continuous regulation of flight direction then becomes the principal means of trajectory control. Consequently, investigations concerned with this specific aspect of flight path management, take on practical significance with respect to evaluating the capabilities of various steering or homing techniques applicable to the guidance of an unpowered gliding vehicle.

Analysis dealing with controlled gliding flight can cover a broad spectrum ranging from the application of optimal control theory[2] to investigations concerned simply with the geometry of the motion.[3] Contrasting the numerical trajectory determinations which follow from treatments such as those developed in Refs. 2 and 3, the results of this study are analytic in scope. A class of closed form solutions are derived from kinematic considerations of a particle maneuvering with constant speed through a uniform velocity field. This representation serves to approximate the behavior of a gliding system executing moderate turns in a constant wind environment. Assuming a specific guidance law, or what amounts to a directional constraint on the airspeed vector eliminates dynamic considerations. Consequently, velocity relationships can be derived directly and solutions for position and time coordinates obtained by integration. These results serve to quantify the performance of the assumed guidance law while providing valuable insight into generalized capabilities.

Analysis

The guidance law or controller used in this formulation is thought of as one which causes the system to maintain a fixed angular orientation between the horizontal projection of its airspeed vector and a radial line connecting it to the intended target. Figure 1 depicts the planar geometry of this motion, introducing the idea of what shall be termed "azimuth homing." The system is idealized as a point mass and under the combined assumptions of constant L/D and moderate turns, only the horizontal coordinates remain coupled. The vertical mode, not shown in Fig. 1, is linear with time t, as in-

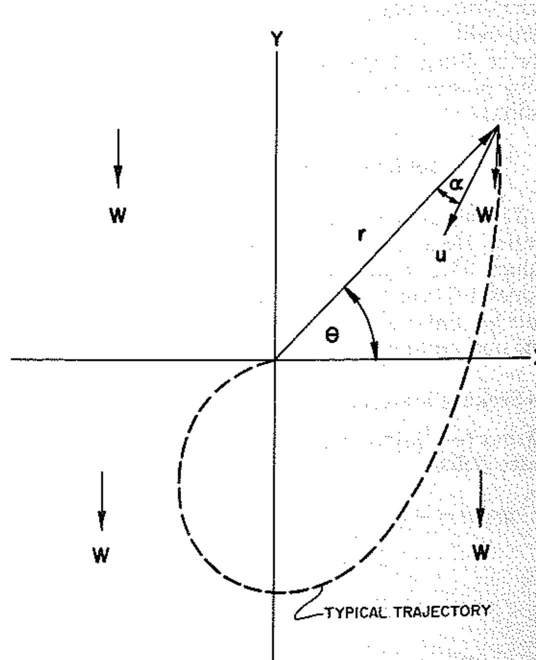

Fig. 1 Kinematics of azimuth homing.

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dicated by Eq. 3. The vector quantity U defines the horizontal component of the total airspeed vector V. U is constant in magnitude, however, its direction may be varied according to control inputs. Azimuth homing, as defined, constrains U to continuously point along the current radial at some fixed angular offset α. The wind, or field velocity, W, is assumed to be steady, to lie entirely in the $x-y$ plane, and to point along the y axis in a negative sense. Motion through the wind field serves to upset kinematic equilibrium and as a consequence, the system is always in a state of maneuver.

It is now possible to derive a vector equation relating the absolute velocity of the system relative to an Earth fixed reference, to the sum of V and W. Expressed in cylindrical coordinates the scalar equations obtained from this vector equality are:

$$dr/dt = -w(\lambda\cos\alpha + \sin\theta) \quad (1)$$

$$(r)d\theta/dt = -w(\lambda\sin\alpha + \cos\theta) \quad (2)$$

$$dz/dt = (-u)/(L/D) \quad (3)$$

where r, θ, and z are the conventional cylindrical coordinate designations.

Trajectory Determination

General Formulation

Separating the time dependency from Eqs. (1) and (2) yields the expression;

$$dr/r = (\lambda\cos\alpha)d\theta/(\lambda\sin\alpha+\cos\theta) - d(\cos\theta)/(\lambda\sin\alpha+\cos\theta) \quad (4)$$

Motivated by physical considerations, solutions to this differential form may be divided into two categories, classified according to their target seeking or target orbiting properties. These classifications occur naturally reflecting the system's capacity to sustain angular motion with respect to the target. This ability is evident from Eq. (2) through investigation of the parameter $\lambda\sin\alpha$. When $|\lambda\sin\alpha|<1$, convergence to a particular ray results. Given sufficient effective wind penetration, the system will seek this stable angular alignment as r approaches zero. Hence, the terminology, "target seeking." Alternatively, cyclic or orbital paths can be described when $|\lambda\sin\alpha|>1$. This velocity condition allows for continuous angular motion about the homing point from all locations in the wind field.

Prior to integrating Eq. (4), a simplified and physically revealing version can be obtained through the variable transformation;

$$\cos\theta = (\cos\beta - \epsilon)/(1-\epsilon\cos\beta) \quad (5)$$

where β is the so called "eccentric anomaly," a nomenclature originating from the development of this equation in orbital mechanics. The magnitude of ϵ defined by Eq. (5) is constant for values between zero and one, and for the purposes of azimuth homing assumes the role of $\lambda\sin\alpha$ or $(1/\lambda\sin\alpha)$ depending upon the application.

Target Seeking Trajectories

$\epsilon=\lambda\sin\alpha$ for $|\lambda\sin\alpha|<1$. Transforming Eqs. (1, 2 and 4) to r, β space yields;

$$dr/dt = -w(1-\epsilon^2)^{1/2}[[(\lambda^2-\epsilon^2)/(1-\epsilon^2)]^{1/2} + \sin\beta/(1-\epsilon\cos\beta)] \quad (6)$$

$$rd\beta/dt = -w(1-\epsilon^2)^{1/2}\cos\beta \quad (7)$$

$$dr/r = [(\lambda^2-\epsilon^2)/(1-\epsilon^2)]^{1/2}(d\beta/\cos\beta) - d(\cos\beta)/\cos\beta(1-\epsilon\cos\beta) \quad (8)$$

Equation (8) is integrated directly to give;

$$r = K\sec\beta(1-\epsilon\cos\beta)[\sec\beta+\tan\beta]^{[(\lambda^2-\epsilon^2)/(1-\epsilon^2)]^{1/2}} \quad (9)$$

where K is an integration constant.

The effect of the β transformation can now be appreciated by allowing ϵ, in Eqs. (5-9), to approach zero while λ is constrained to remain finite. In terms of physical coordinates this defines the special case of radial homing.[4] A comparison between Eqs. (6-9) for arbitrary ϵ, with those produced when $\epsilon=0$ reveals their similarity, particularly regarding Eqs. (7) and (9). It can be concluded, based on this similarity, that azimuth homing, constrained such that $|\lambda\sin\alpha|<1$, is a general form of radial homing when viewed in $r-\beta$ space. Therefore, the terminal state capabilities of radial homing derived from the physical plane establishes these characteristics for an entire azimuth homing family. The significant features of these trajectories are summarized as follows. Launched at sufficient altitude, an azimuth homing system will reach the target or homing point prior to impact provided it has the ability to penetrate the wind, i.e., when the systems generalized wind penetration parameter $[\lambda^2-\epsilon^2/1-\epsilon^2]^{1/2}>1$. At target arrival the absolute or resultant velocity vector will be aligned in a negative sense along the $\beta=-\pi/2$ ray. The arbitrary alignment of the wind velocity with the y axis, which assisted in the formulation of the basic theory, has no bearing on these results. Any other axis selection merely constitutes a rotation of the resulting trajectory relative to these primary coordinates producing no effect on the end state. The terminal characterisitics, $(r=0,\ \beta=-\pi/2)$, are also essentially independent of initial homing conditions determined by the algebraic sign of ϵ.

Orbital Trajectories

$\epsilon=(1/\lambda\sin\alpha)$ for $|(1/\lambda\sin\alpha)|<1$. Transforming Eqs. (1), (2) and (4) to $r-\beta$ space yields;

$$dr/dt = (-w/\epsilon)[1-\epsilon^2]^{1/2}[b/(1-\epsilon^2)^{1/2} + \epsilon\sin\beta/(1-\epsilon\cos\beta)] \quad (10)$$

$$rd\beta/dt = (-w/\epsilon)[1-\epsilon^2]^{1/2} \quad (11)$$

and, $dr/r = (b/[1-\epsilon^2]^{1/2})d\beta - d(\epsilon\cos\beta)/(1-\epsilon\cos\beta)$ (12)

where $b=\text{ctn}\alpha$. Equation (12) is integrated directly to give

$$r = G(1-\epsilon\cos\beta)e^{(b/[1-\epsilon^2]^{1/2})\beta} \quad (13)$$

where G is an integration constant.

The net behavior of Eq. (13) can be determined by considering separately, the properties of the trigonometric and exponential factors which combine to form r. The quantity $(1-\epsilon\cos\beta)$ is identified as a polar representation describing one of the Limacon's of Pascal, thereby defining a closed curve in the $r-\beta$ plane. The equivalent figure in physical coordinates is an ellipse and comprises the entire trajectory when α is allowed to assume either of its extreme values (i.e. when $\alpha=\pm\pi/2$). Overall, angular motion relative to the target can be positive or negative depending upon the arbitrary sign of the radial offset angle α. Regardless of the sense of change in β or θ, the argument of e remains negative. Consequently, the exponential terms always acts as a damping factor, continually suppressing radial excusions in successive cycles about the focal point. Given the nature of the functions defining Eq. (13), the trajectories so generated may be properly termed "Elliptic Spirals."

Launch Criteria

It is now desirable to obtain solutions in terms of the time or altitude coordinate. Such relationships will be necessary to identify spatial locations compatible with the trajectories previously obtained. Returning to Eqs. (2) and (3),

$$dt = [(-L/D)/u]dz = [(-1/w)rd\theta]/[\lambda \sin\alpha + \cos\theta] \quad (14)$$

where r is now a known function of θ, or β.

Solutions to Eq. (14), which exist at least in principle, can now be attempted for both the target seeking and target orbiting cases.

Time Integral

Two forms of Eq. (14) are generated by the respective utilization of Eqs. (7) and (9) or Eqs. (11) and (13). The target seeking form of Eq. (14) is:

$$(L/D)(z_i/u) = (-Kw[1-\epsilon^2]^{1/2} [\int \sec^2\beta(1-\epsilon\cos\beta)(\sec\beta+\tan\beta)^{[(\lambda^2-\epsilon^2)/(1-\epsilon^2)]^{1/2}} d\beta] \quad (15)$$

The target orbiting version of Eq. (14) becomes:

$$(L/D)(z_i/u) = -G\epsilon/w[1-\epsilon^2]^{1/2} \int (1-\epsilon\cos\beta)e^{b/[1-\epsilon^2]^{1/2}} d\beta \quad (16)$$

where in both cases integration on altitude passes from the initial point z_i to the ground plane ($z=0$). Radial and angular coordinates proceed from the initial state r_i, θ_i to final positions r, θ which now specify the systems location at impact. Both Eqs. (15) and (16) can be integrated by parts producing the following solution which holds for either circumstance.

$$(L/D)(z_i\cos\alpha/r_i) = [\lambda/(\lambda^2-1)]\,[\lambda-\sin(\theta_i-\alpha)]\,[1-(r/r_i)(\lambda-\sin(\theta-\alpha))/(\lambda-\sin(\theta_i-\alpha))] \quad (17)$$

where the position coordinate r is given by Eq. (9) for trajectories in the target seeking domain or by Eq. (13) for the target orbiting case.

Release Path Definition

Given a specified launch or initial altitude z_i, system parameters in terms of L/D, α, and λ which reflects wind properties as well, the following physical interpretation can be attached to Eq. (17). Allowing for sufficient performance in terms of either wind penetration ability, or the potential to execute multiple orbits about the homing point, r can be made to approach zero in Eq. (17). Then, relative to a known nominal wind direction, the curve so described by this equation defines the locus of points (z_i, r_i, θ_i) from which an azimuth homing trajectory will terminate precisely on target. Equation (17), with $r=0$ the final state, identifies this release path or launch curve as an ellipse. It follows from these developments, that trajectories beginning inside the ellipse will arrive too early, (i.e. an excess altitude condition), while those initiated outside will fall short.

Summary and Conclusions

Analysis of a particular guidance technique applicable to gliding vehicle flight path control has been presented. The development has proceeded through three phases as follows: 1) formulation of kinematic behavior, leading to the derivation of basic relationships, 2) mathematical transformation of variables with subsequent solution to the governing differential equations, and finally, 3) classification and interpretation of results according to the dictates of key parameters.

The azimuth homing guidance technique has been shown to possess desirable performance particularly regarding the ability to either maintain position relative to a homing point in a captured spiral orbit, or to achieve absolute target convergence in the horizontal plane. Since guidance is derived exclusively through management of flight direction, consistent accuracy will not be possible in general circumstances. However, a scheme employing this form of guidance augmented by computations utilizing position measurements could be used to control the entire flight. The radial offset angle (α), being capable of discrete variation, can be utilized so as to adjust the trajectory to account for uncontrollable changes in wind velocity. The wind penetration parameter (λ) may also be considered as a control parameter assuming some L/D modulation were possible. Theoretically, and within the spatial locations where terminal solutions to the azimuth homing problem exist, the entire landing state could be controlled.

References

[1] Slayman, R. A., Bair, H. W., Rathbun, T. W., "500-Pound Controlled Airdrop Cargo System," GER-13801, Sept. 1970, Goodyear Aerospace Corp., Akron, Ohio.

[2] Pearson, A. E., "Optimal Control of a Gliding Parachute System," TR-73-30-AD, Aug. 1972, USA Natick Labs., Natick, Mass.

[3] Goodrick, T. F., "Wind Effect on Gliding Parachute Systems with Non-Proportional Automatic Homing Control, TR-70-28-AD, Nov. 1969, USA Natick Labs., Natick, Mass.

[4] Murphy, A. L., Jr., "Trajectory Analysis of a Radial Homing Gliding Parachute In a Uniform Wind," TR-73-2-AD, Sept. 1971, USA Natick Labs., Natick, Mass.