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Heat Transfer and the Killing of Bacteria in Thermal Sterilization of Meat Roll

CHIA PING WANG

Research Physicist,
Radiation Sources Group

ARI BRYNJOLFSSON

Chief
Radiation Preservation of Food Division,
Food Engineering Laboratory,
U. S. Army Natick Research & Development Command,
Natick, Mass.

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ABSTRACT

Exact relation has been obtained between temperature in every point of a meat roll and the time. The theoretical results so obtained were compared with measurement. This relation was then used to determine the "integral" survival curve for *C. botulinum* spores as a function of time in different parts in the roll. For sterilization at 121°C of a very long meat roll, with a 5 cm radius, it is found that a heating time of about 4 hours is needed in order to reduce the survival fraction of *C. botulinum* spores to 10^{-12} . For a roll of the same diameter, but of a length equal to one-half of the diameter, the required heating time is reduced to about 90 minutes (37% of 4 hours). The thermal diffusivity for the beef roll found from this study is about $1.3 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, the thermal conductivity, $0.4 \text{ watt M}^{-1} \text{ }^\circ\text{C}^{-1}$, and the effective surface resistance number (or conduction Nusselt number or Biot number) about 7.

NOMENCLATURE

a = Radius of the meat roll in cm.
 c = Heat capacity in J.g^{-1} .
 D = Decimal reduction time in s.
 E_a = Arrhenius activation energy in kcal.mol^{-1} for inactivating a unit or a molecule that is essential for the survival of a bacterium.
 f_h = Time required for product to traverse one log cycle of temperature.
 $J_0(x_n)$ = Bessel function of the first kind of order zero.

$J_1(x_n)$ = Bessel function of the first kind of order one.
 k = Reaction rate constant in s^{-1} for inactivation of a microorganism.
 N = The number of bacteria survived at time t .
 dN = Infinitesimal change in N .
 N_0 = Initial value of N .
 R = The universal gas constant in J.mol^{-1} .
 T = Temperature in Kelvin.
 T_i = Initial temperature in Kelvin of the entire meat.
 T_o = Heating temperature in Kelvin outside the meat roll.
 $T(t)$ = Temperature in Kelvin at any arbitrary point of the meat roll as a function of time.
 x_n = $a \alpha_n$ = Roots of Eq. (10).
 z = The z value in $^\circ\text{C}$.
 α_n = x_n/a = Roots of Eq. (10)/radius of the cylinder.
 δ_{gas} = Thickness of the gas (air) film adhered onto the skin of the meat roll in cm.
 κ = Thermal diffusivity of meat in $\text{cm}^2 \cdot \text{s}^{-1}$.
 λ = Thermal conductivity of meat in $\text{J} \cdot \text{s}^{-1} \cdot \text{cm}^{-1} \cdot \text{ }^\circ\text{C}^{-1}$.
 λ_{gas} = Thermal conductivity of the heating gas (air) outside the meat roll in $\text{J} \cdot \text{s}^{-1} \cdot \text{cm}^{-1} \cdot \text{ }^\circ\text{C}^{-1}$.
 v = Effective conduction Nusselt number or Biot number.
 ρ = Density of meat in g cm^{-3} .
 θ = $T - T_o$
 θ_i = $T_i - T_o$

MATHEMATICAL INTRODUCTION FOR THE EVALUATION OF THE SURVIVAL FRACTION

It is well known (1) that for quite a large class of microorganisms the logarithmic survival fraction, $\ln N/N_0$, for thermal sterilization at a specific temperature T is a linear function of the sterilization time t. This relation is explained by laws of thermodynamics which show that the frequency by which a subsystem (an atom or a molecule) has an energy E_a above the average thermodynamic energy is proportional to the exponential function $\exp(-E_a/RT)$, the Boltzmann factor, where E_a is the Arrhenius activation energy, T the temperature in °K, and R, the universal gas constant. That is:

$$dN = -kNdt = -C \cdot [\exp(-E_a/RT)] \cdot N \cdot dt \quad (1)$$

In Eq. (1) we think of E_a as the activation energy for inactivating a unit or a molecule (for instance a DNA "molecule") that is essential for survival of the bacterium. Each time an essential unit is inactivated, a bacterium is killed. The number dN of bacteria killed in a short time dt is then proportional to the total number N of bacteria and to the Boltzmann factor $\exp(-E_a/RT)$. C is a constant and is a measure of the absolute frequency of energy transfer E_a .

It follows from the reaction rate equation (1) by integration that:

$$\ln \frac{N}{N_0} = -C \int_0^t \exp(-E_a/RT) dt \quad (2)$$

If in Eq. (2) T is constant in time then:

$$\ln \frac{N}{N_0} = -[C \exp(-E_a/RT)]t = -kt = -2.303 t/D \quad (3)$$

$$\text{or } N = N_0 \cdot \exp(-kt) = N_0 \exp(-2.303 t/D)$$

For a large number of microorganisms, the semi-logarithmic plot of the survival fraction N/N_0 against t is not a straight line, but has a "shoulder". In such a case, the rate constant k is time-dependent even though the temperature is held constant. In the following, we prove in two ways that the form of Eq. (2) holds for any function F(T,t) on the right-hand side of Eqs. (1) or (2).

Proof I.

If at constant temperature T,

$$\frac{dN}{dt} = -C \cdot F(T,t) \cdot N \quad (1a)$$

then we have the "differential law" similar to Eq. (1).

Hence, when T varies with t, we have, by integrating Eq. (1a),

$$\ln(N/N_0) = -C \cdot \int_0^t F(T(t), t) dt \quad (2a)$$

For those who prefer to take Eq. (1a) as an assumption rather than a fundamental law, we prove Eq. (2a) without assuming Eq. (1a) in Proof II below.

Proof II.

If at constant temperature T, the logarithmic survival fraction is given by

$$\ln(N/N_0) = -C \cdot F(T,t) \cdot t \quad (1b)$$

then for a small time interval Δt at time t_1 , there will be a mean effective temperature T_1 and a survival fraction N_1/N_0 , such that

$$\ln(N_1/N_0) = -C \cdot F(T_1, t_1) \cdot \Delta t$$

Similarly, for Δt at (t_2, T_2) following the first time interval Δt at t_1 , and so on,

$$\ln(N_2/N_1) = C \cdot F(T_2, t_2) \cdot \Delta t$$

$$\ln(N_3/N_2) = -C \cdot F(T_3, t_3) \cdot \Delta t$$

$$\vdots$$

$$\ln(N_n/N_{n-1}) = -C \cdot F(T_n, t_n) \cdot \Delta t$$

Summing the above,

$$\ln(N_n/N_0) = -C \cdot \sum_{i=1}^n F(T_i, t_i) \cdot \Delta t$$

Hence

$$\ln(N/N_0) = -\lim_{\Delta t \rightarrow 0} C \cdot \sum_{i=1}^{\infty} F(T_i, t_i) \cdot \Delta t$$

$$= -C \int_0^t F(T(t), t) dt \quad (2a)$$

which is Eq. (2a).

Thus, we can use Eq. (2), or more generally Eq. (2a), with perfect confidence in evaluating the survival fraction N/N_0 .

From Eq. (2a), N/N_0 can be accurately determined if $F(T,t)$ as a function of the temperature T and the time t is found, and the temperature $T(t)$ of the specimen as a function of the time t is known.

In the following section, we show how the function $T(t)$ for a long meat roll is derived from the equations of heat transfer. The equations of heat transfer are based on well-established physical laws; and their solutions, though complicated and lengthy, have been obtained for a great number of cases by workers in the various fields (2,3). In the following, we outline our mathematical solution for the case of a long meat roll. As will be seen below, the complicated situation of conductivity discontinuity of the meat roll skin (the casing), of the convective heating and of the radiation exchange at the surface, can all be taken into account by a combined single statement of the boundary condition.

TEMPERATURE DISTRIBUTION IN A LONG MEAT ROLL

The heat conduction equation for an infinitely long cylinder of radius a , is

$$\frac{\partial T}{\partial t} = \kappa \left\{ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right\}, \text{ for } 0 < r < a \quad (4)$$

where κ is the "thermal diffusivity"

$$\kappa = \frac{\lambda}{\rho \cdot c} = \frac{\text{Thermal conductivity}}{\text{density} \cdot \text{specific heat}} \quad (4a)$$

Initially the entire meat roll is at temperature T_i . Then suddenly the meat roll is exposed to the temperature T_o and the outside temperature is maintained at T_o during the process. The temperature T at any point in the meat roll will change with time. To simplify the equations, we will use a new variable θ where

$$\theta = T - T_o \quad (5)$$

Eq. (4) then becomes

$$\frac{\partial \theta}{\partial t} = \kappa \left\{ \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right\} \quad (6)$$

The initial value of θ at any point of the meat roll is:

$$\theta_i = T_i - T_o \text{ for } t \leq 0 \quad (7)$$

At the boundary $r = a$, there will be in practice an interface (layer) or transition layer in which the temperature is changing from T_o to the actual temperature at the surface of the roll. At the surface, the boundary condition of heat flowing into the roll being equal to the heat flowing out from the layer across the surface of the roll, leads to

$$-\frac{\partial \theta}{\partial r} = \frac{h}{\lambda} \theta, \text{ for } r = a \quad (8)$$

where $h = \frac{\lambda_{\text{gas}}}{\delta_{\text{gas}}}$, the conductivity of the gas (air), divided by the thickness of the gas (air) film, is the coefficient of heat transfer due to gas film only.

Due to radiation at the surface, h actually would be equal to $\lambda_{\text{gas}}/\delta_{\text{gas}}$ plus some constant (due to radiation). Also, because of the skin (the casing) of the roll, we have actually two transition layers instead of one at the surface, the solid skin and the gas film adhered onto it. It can be shown that all these three factors can be combined into a single effective coefficient in front of θ in Eq. (8). Henceforth, h/λ of Eq. (8) will stand for this effective coefficient.

Eq. (6) with boundary condition of the form of Eq. (8), i.e. $-\frac{\partial \theta}{\partial r}$ being proportional to θ , has been

solved with the usual technique of the separation of variables. Here we use Carslow and Jaeger's notations (2) when applicable. The solution takes the form

$$\theta = \sum_{n=1}^{\infty} A_n J_0(\alpha_n r) \cdot e^{-\kappa \alpha_n^2 t} \quad (9)$$

where $\alpha_n = x_n/a$ are roots of

$$x_n \cdot J_1(x_n) = \frac{h}{\lambda} a \cdot J_0(x_n) \quad (10)$$

and

$$A_n = \frac{2\theta_i}{x_n} \cdot \frac{J_1(x_n)}{J_0^2(x_n) + J_1^2(x_n)} \quad (11)$$

In Eqs. (9), (10), and (11), J_0 and J_1 are Bessel functions of orders zero and one respectively.

Hence, from Eqs. (5) and (7),

$$T - T_o = 2(T_i - T_o) \cdot \sum_{n=1}^{\infty} \frac{1}{x_n \left\{ \left(\frac{x_n}{v}\right)^2 + 1 \right\} J_1(x_n)}$$

$$J_0(\alpha_n r) \cdot e^{-\kappa \alpha_n^2 t} \quad (12)$$

where

$$x_n = \alpha_n a \quad (13)$$

$$v = \frac{h}{\lambda} a = \begin{matrix} \text{effective conduction} \\ \text{Nusselt number or} \\ \text{Biot number} \end{matrix} \quad (14)$$

Had we considered only the gas film at the surface, h in Eqs. (14) and (8) would be $\lambda_{\text{gas}}/\delta_{\text{gas}}$, and v of Eq. (14) would be the "conduction Nusselt number" (3), or the "Biot number", which is to be distinguished from the "convection Nusselt number", the latter being equal $\frac{h}{\lambda_{\text{gas}}} \cdot a$.

In our case, $\left(\frac{h}{\lambda}\right)$ is the combined effective coefficient of three factors, the gas film, the solid skin, and the surface radiation. But, it would be meaningful to express the combined effect in terms of an equivalent resistive film at the surface. Thus, the combined constants $\frac{h}{\lambda} a$ of Eq. (14) is the equivalent or effective conduction Nusselt number or Biot number.

In Eq. (12) there are thus two constants or parameters, the thermal diffusivity κ and the effective Nusselt number v which relates to the combined thermal resistance of the surface. They may be determined separately or found by fitting Eq. (12) with measured values of T as a function of time. We carried out a computer search for the ranges of these two parameters, κ and v , and found that the thermal diffusivity κ lies somewhere around 1.25×10^{-3} to $1.35 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, and the effective Nusselt number v , around 6 to 8, for the beef rolls processed in these experiments.

Fig. 1 is a computer plot of Eq. (12) as a function of time for $r = 0, 0.1 a, 0.2 a \dots 1.0a$, with $v = 6$, $\kappa = 1.35 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, and $a = 5 \text{ cm}$. The heating and the initial temperatures are 121.1°C and 21°C respectively.

Fig. 2 is a computer plot of the temperature distribution within the roll for $t = 5, 10, 15 \dots$ 240 minutes, i.e. the plot of Eq. (12) as a function of r for $t = 5, 10, 15 \dots, 240$ minutes, again with $v = 6, \kappa = 1.35 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}, a = 5 \text{ cm}$, and the same heating and initial temperatures as Fig. 1.

CALCULATIONS OF E_a, C AND THE INTEGRAL BACTERIA SURVIVAL FRACTION

We can now put our function T of Eq. (12) into the "bacteria survival integral" of Eqs. (2) or (2a) to calculate the survival fraction N/N_0 of the bacterium. In this paper, we carried out the calculations for *Cl. botulinum* spores for which we assume the reaction rate constant k to be given by the exponential function $\exp(-E_a/RT)$ rather than the general function $F(T(t), t)$ which may represent any empirical function or other theoretical function such as Eyring's.

The calculations for other microorganisms will be exactly the same.

The integrand of Eq. (2) after substituting T from Eq. (12) looks quite complicated. We have written computer programs to carry out the computation. But, before we could actually carry out the computation, the two constants C and E_a in Eqs. (2) and (1) have to be determined.

Determination of E_a from the z Value

We show in some details below the relationship between the z -value, the temperature T , and the Arrhenius activation energy E_a because there seems to have been some misunderstanding expressed by some of the reviewers about the functional relationship of Eq. (1) and the equations (given below) derived from it to express E_a in terms of the z -value.

As remarked in the first section, the reaction rate equation, Eq. (1),

$$\frac{dN}{dt} = -kN = -C \cdot [\exp(-E_a/RT)] \cdot N \quad (1)$$

means that the temperature dependence of k is through T in the exponential function $\exp(-E_a/RT)$ only, where E_a is a constant independent of T . Otherwise, k will have to be expressed by the general function $F(T(t), t)$ of Eq. (1a).

Now for bacterium of number N at temperature T_1 , the reaction rate, from Eq. (1), is

$$\left(\frac{dN}{dt} \right)_{T_1} = -C \cdot [\exp(-E_a/RT_1)] \cdot N \quad (15)$$

Similarly, the reaction rate at temperature T_2 for the same bacterium of the same number N , is

$$\left(\frac{dN}{dt} \right)_{T_2} = -C \cdot [\exp(-E_a/RT_2)] \cdot N \quad (16)$$

Thus, from Eqs. (15) by (16)

$$\begin{aligned} \frac{\left(\frac{dN}{dt} \right)_{T_1}}{\left(\frac{dN}{dt} \right)_{T_2}} &= \exp \left[-\frac{E_a}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \right] \\ &= \exp \left[-\frac{E_a}{R} \frac{T_2 - T_1}{T_1 T_2} \right] \quad \text{--- (17)} \end{aligned}$$

Now if we choose the temperature T_2 such that the rate of inactivation is changed by a factor of 10 from that at T_1 , then

$$\frac{\left(\frac{dN}{dt} \right)_{T_1}}{\left(\frac{dN}{dt} \right)_{T_2}} = 10 \quad (18)$$

Equating Eqs. (18) and (17) gives

$$E_a = \frac{(\ln 10) \cdot R \cdot T_1 T_2}{T_1 - T_2} = \frac{(\ln 10) \cdot R \cdot T_1 T_2}{z} \quad (19)$$

where, by definition, $T_1 - T_2$, the temperature change as specified, is the z -value.

In other words, the values of the two slopes, Eqs. (15) and (16), at the two temperatures T_1 and T_2 , determine the unknown constant E_a in these equations, and is given in Eq. (19).

We note that Eq. (19), contrary to what one reviewer remarked, by no means expressed the temperature dependence of E_a . The appearance of $T_1 T_2$ on the right-hand side simply means that, if the reaction rate equation Eq. (1) is strictly true, then there is a temperature dependence of the z -value such that

$$\frac{T_1 T_2}{z} \text{ is a constant} \quad (20)$$

for any two temperatures T_1 and T_2 giving a rate change by a factor of 10.

We could use, however, Eq. (20) to test whether the Arrhenius activation energy E_a , has only one definite value for a particular bacterium.

The evaluation of E_a is straightforward once the z -value for a particular bacterium is known. For *Cl. botulinum* spores, representative values for z are in the range $z = 10.4 \pm 0.8^\circ\text{C}$ at 121.1°C (4). From Eq. (19), we have then for *Cl. botulinum* spores,

$$\begin{aligned} E_a &= \frac{(\ln 10) \cdot 394.26 \cdot 384.26 \cdot 1.987 \cdot 10^{-3}}{10.4} \\ &= 66.6 \pm 5.4 \text{ kcal. mol}^{-1} \quad (21) \end{aligned}$$

Determination of the Constant C

For *Cl. botulinum* spores, we will, in this paper, use for 12D, the conservative F_0 -value of 3.5 min (4), which means that heating at 121.1°C for 17.5 sec. reduces the number N by one order of magnitude. Thus, from Eq. (3), for *Cl. botulinum* spores,

$$[C \cdot \exp(-E_a/RT)] \cdot 17.5 = \ln 10$$

giving

$$C = \frac{2.3026}{17.5} \times e^{85.076} \\ = 1.17 \cdot 10^{36} \text{ s}^{-1} \quad (22)$$

Calculation of the Bacteria Survival Integral and the Integral Survival Curves

The calculation of the constant C is indeed very simple as given in the above section. We note, however, that C is a very large number, which, in turn, means that the "Bacteria Survival Integral" in Eq. (2) will give very small numbers. Certain procedures have been taken to overcome the computer underflow when computing this integral.

Fig. 3 is the plot of the "integral" survival curves for *C. botulinum* spores in an infinite cylinder with radius $a = 5$ cm. The calculations are made using Eq. (2) in which T is given by Eq. (12), for $r = 0, 0.1a, 0.2a, \dots, 1.0a$, with $v = 6$ and $\kappa = 1.35 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$. The heating and the initial temperatures, as before, are 121.1°C and 21.1°C , respectively.

It is seen from the survival curve for $r = 0$ that in order to reduce the survival fraction of *C. botulinum* spores to 10^{-12} of the initial spore number, 3.5 hours heating is needed for a long beef roll of radius = 5 cm. This value of heating time is obtained with $v = 6$ and $\kappa = 1.35 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$.

Because there are spreads of the values of both κ and v for any meat roll, there will be corresponding spreads of these integral survival curves. Fig. 4 shows the spreads of the integral survival curve for $r = 0$ due to the spreads of κ and v . Curve 1 is the plot for $r = 0$, with $v = 6$ and $\kappa = 1.35 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, and curves 2 and 3 are similar plots for $r = 0$ with the same value of v but κ is set equal to 1.45×10^{-3} and $1.25 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, respectively. For $N/N_0 = 10^{-12}$, the spread is about 12 minutes either way.

The spread due to the variation of the value of v can be seen by comparing curve 1 and curves 4 and 5. It is seen that when v is decreased from 6 to 4, while κ is kept at $1.35 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, the heating time for $N/N_0 = 10^{-12}$ increases from about 3-1/2 hrs to about 4.

The spread of the values E_a from 61.2 to 72 (see Eq. (21)) will also produce a spread of the survival curves. For $N/N_0 = 10^{-12}$, this spread is about 8 minutes either way from the curve with $E_a = 66.6 \text{ kcal mol}^{-1}$.

The above calculations and the values arrived at are for a long meat roll. For a beef roll of the same diameter (10 cm) but with the length of say 7.5 cm, the required heating time for reduction factor 12D, as estimated from the case of zero surface heat resistance (no skin, no surface film and no surface radiation), would be reduced to about 57%, and with the length of 5 cm, to about 37% of the heating time valid for infinitely long rolls. The exact calculations for this case are subjects of our next investigation.

DISCUSSION

In the second section, by fitting the heating curve (temperature wave) for the center of the beef roll, we found that the thermal diffusivity of the roll is around $1.30 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, and v about 7.0. Cohen and Wierbicki (5) have measured the f_h value for beef rolls in cans, i.e., the time required for the time-temperature graph to traverse one log cycle of temperature. From the measured f_h value, they deduced, disregarding surface resistance, that the thermal diffusivity κ for beef is about $1.25 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, in good agreement with our value from fitting the heating curve of the beef roll.

In the preceding paper (6), we have found that the specific heat of beef in unfrozen state with 60% water content is $3.10 + 0.39 \text{ J} \cdot \text{g}^{-1} \cdot ^\circ\text{C}^{-1}$ ($0.74 + 0.09 \text{ cal} \cdot \text{g}^{-1} \cdot ^\circ\text{C}^{-1}$). The range of the values of thermal conductivity of beef with 60% water content is about $(4.0 \pm 0.6) \cdot 10^{-3} \text{ J} \cdot \text{s}^{-1} \text{ cm}^{-1} \cdot ^\circ\text{C}^{-1}$ as reported by Sweat (7) who analyzed 110 thermal conductivity data above freezing compiled by Kostaropoulos (8) and Vachon et al (9). We can then deduce the value of the κ using Eq. (4a).

$$\kappa = \frac{\lambda}{\rho \cdot c} \\ = \frac{4 \cdot 10^{-3}}{1 \cdot 3.1} = 1.29 \cdot 10^{-3} \text{ cm}^2 \text{ s}^{-1}$$

in good agreement with the experimental value reported above.

CONCLUSION

The equation for the "integral" survival fraction of a microorganism is shown to be valid for any form of temperature dependence of the reaction (i.e. killing) rate constant, whether exponential or not. When the survival curve at constant temperature and the time dependence of the temperature of a meat sample are known, then the integral survival fraction can be predicted for the sample from the equation.

The thermal diffusivity κ of beef deduced here from the heating curve of the beef roll and the specific heat obtained in the preceding paper give an overall consistency picture of the seemingly unrelated thermal data of beef reported by the various workers. For beef with 60% water content, κ is about $1.3 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, and λ , the thermal conductivity, is about $0.4 \text{ watt} \cdot \text{M}^{-1} \cdot ^\circ\text{C}^{-1}$. The effective conduction Nusselt number v or Biot number is about 7 for the beef rolls processed in these experiments.

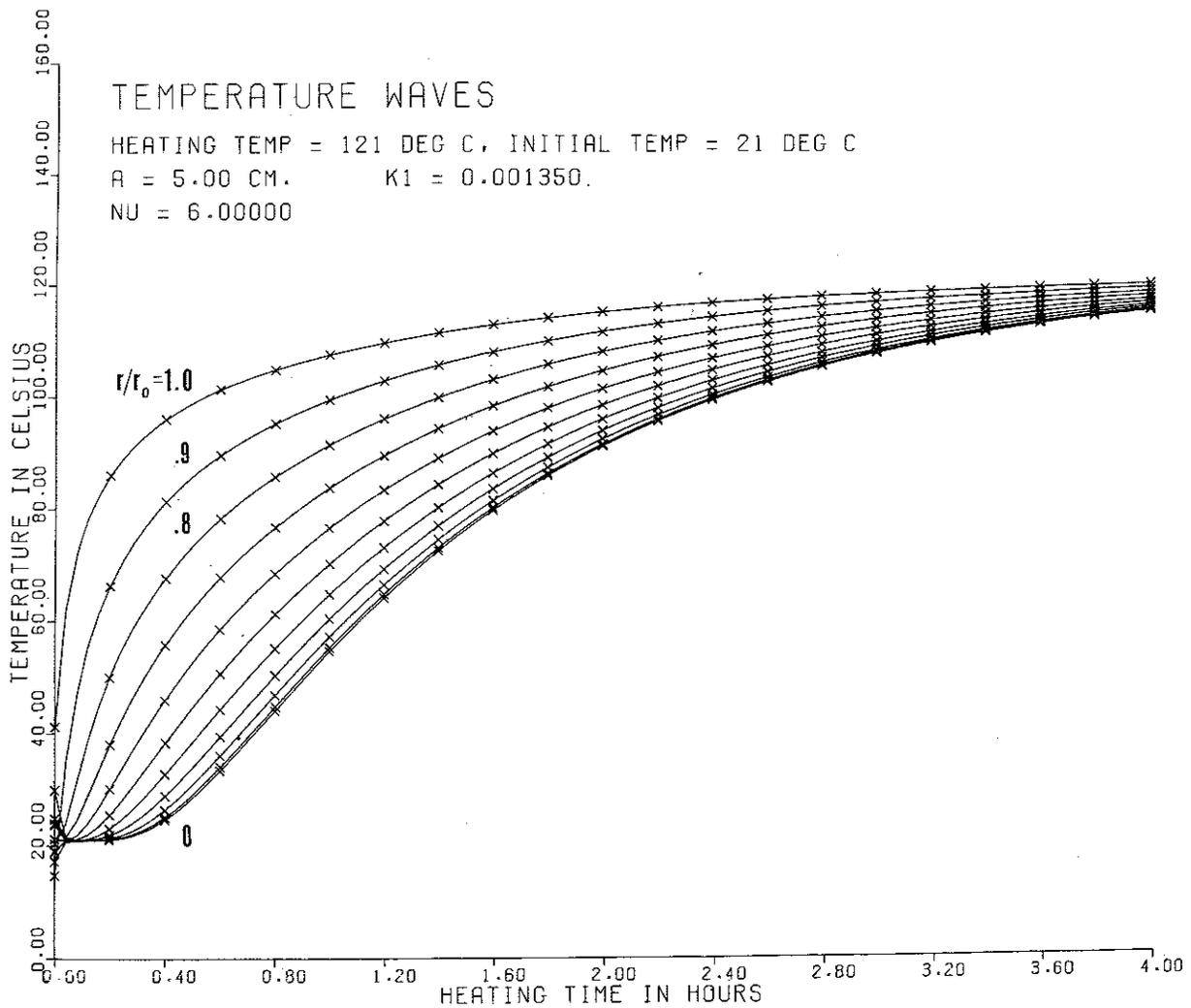


Fig. 1 Computer plot of temperature waves in a long meat roll at radial distances $r = 0, 0.1a, 0.2a, \dots, 1.0a$, starting from the lowest curves. The crosses are for the purposes of providing the 0.2 hr time marks. $K1$ is the thermal diffusivity κ , and NU is the effective conduction Nusselt number ν or Biot number. $r_0 \equiv a$.

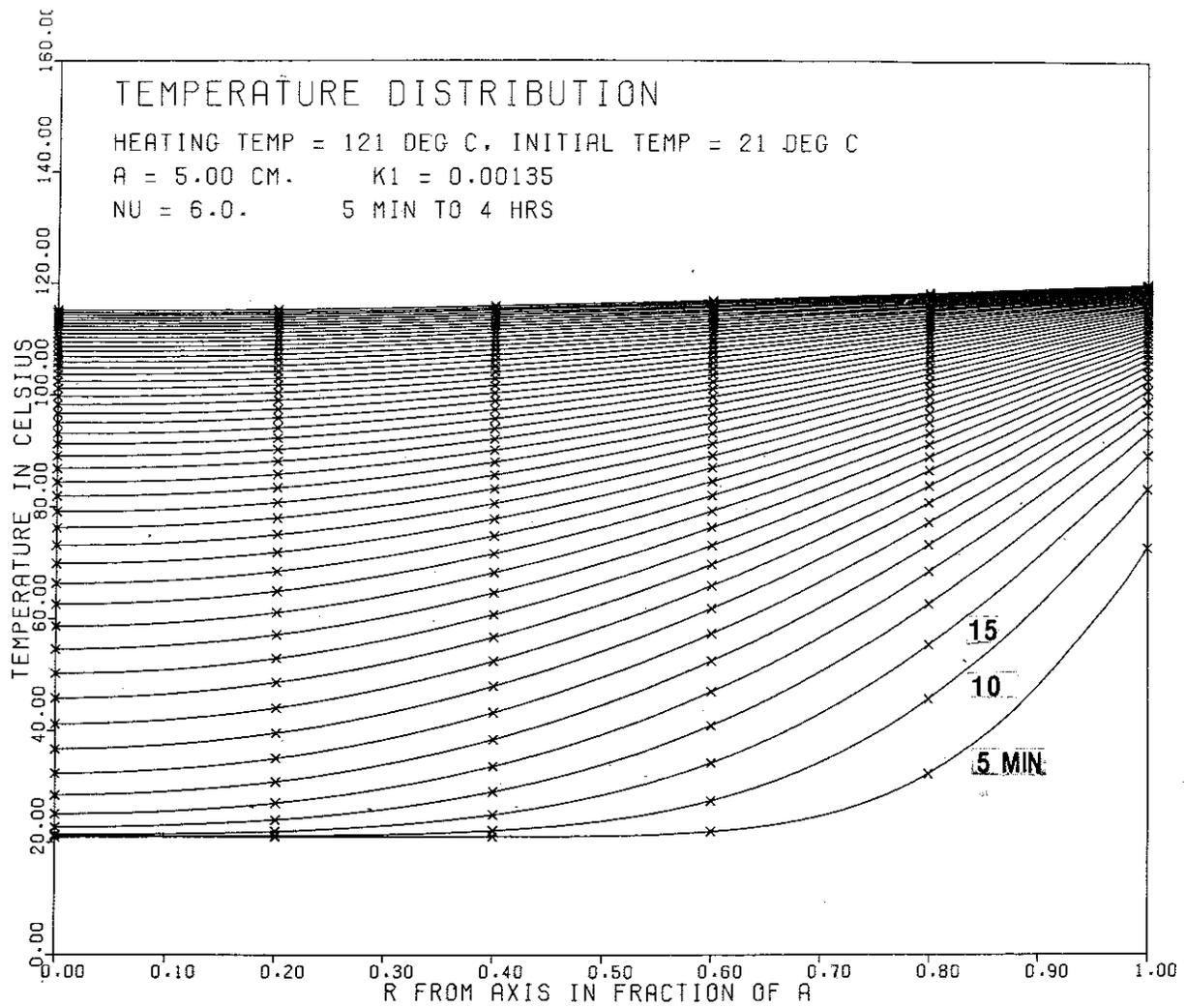


Fig. 2 Computer plot of the temperature distribution in a long meat roll at 5 minutes interval, starting from the lowest curve. The crosses are for the purposes of providing the radial distance markings. $K1$ is the thermal diffusivity κ , and NU is the effective conduction Nusselt number ν or Biot number.

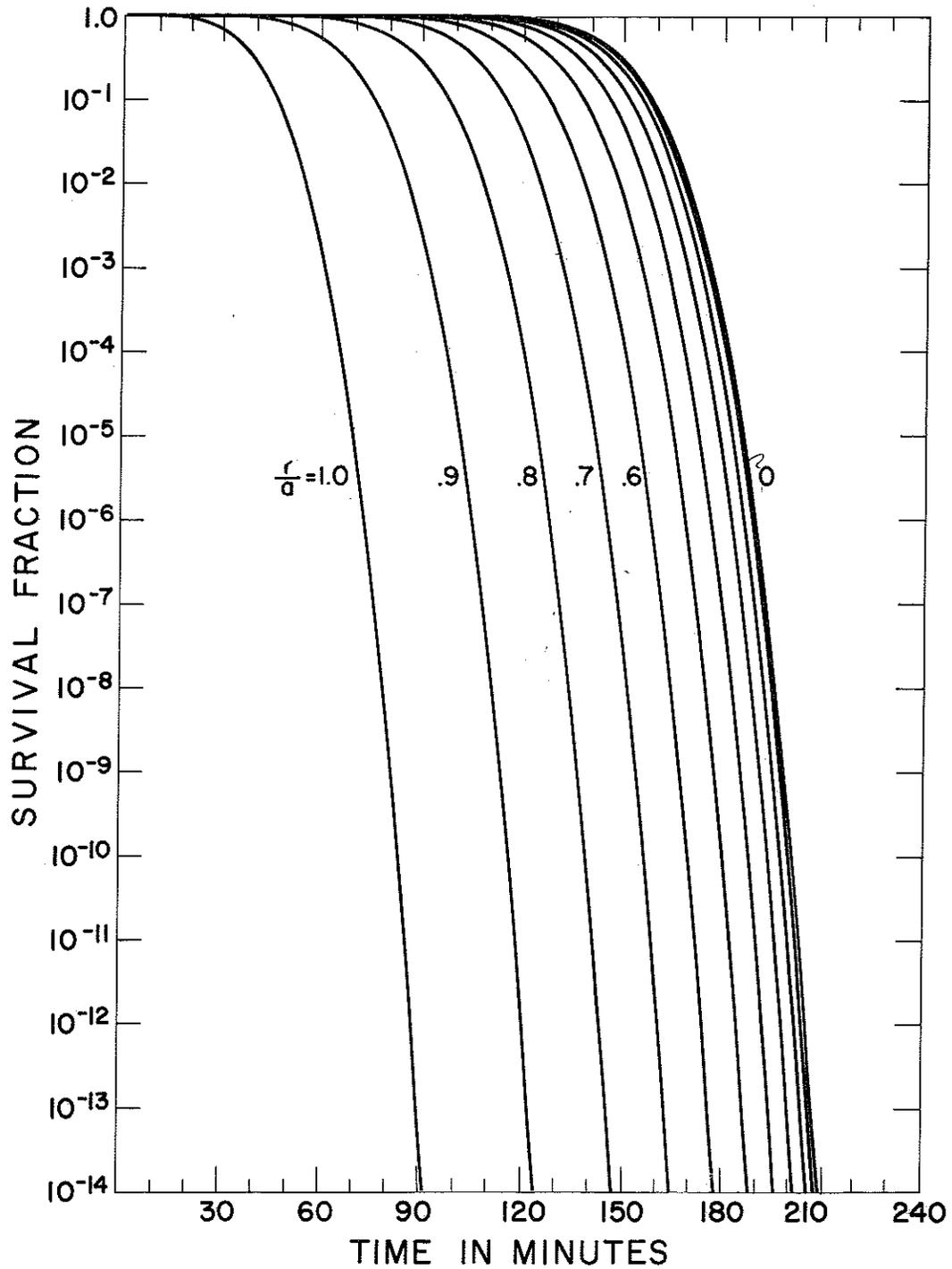


Fig. 3 Integral survival curves for *Cl. botulinum* at the various radial distances in a long meat roll of radius $a = 5$ cm. The values of the parameters are: $\nu = 6.0$, and $\kappa = 1.35 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$. The heating temperature $T_0 = 121^\circ\text{C}$ and the initial temperature $T_1 = 21^\circ\text{C}$.

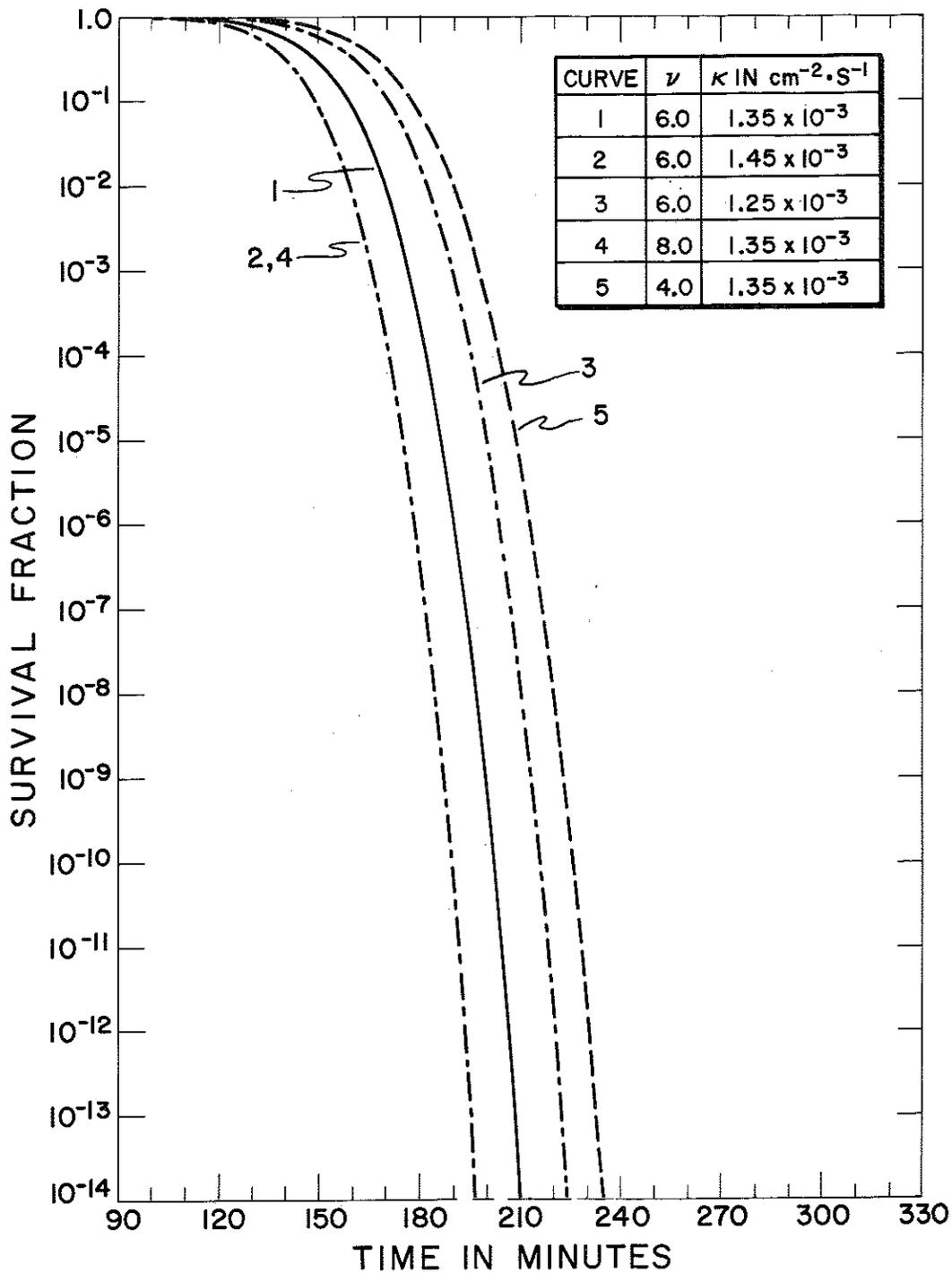


Fig. 4 The spread of the integral survival curves for Cl. botulinum spores at the axis of a long meat roll due to variation of the values of thermal diffusivity κ and effective conduction Nusselt number ν or Biot number. The radius a of the roll is 5 cm. The heating temperature $T_0 = 121^\circ\text{C}$ and the initial temperature $T_i = 21^\circ\text{C}$. For a roll of the same diameter and thickness equal to one-half the diameter, the $12D$ value for each curve is approximately reduced to 37% of the value for infinitely long roll.

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