R93-50

DYNAMICAL SYSTEMS ANALYSIS OF AN AERODYNAMIC DECELERATOR: BIFURCATION TO DIVERGENCE AND FLUTTER

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INTRODUCTION

The prediction of a circular decelerator's behavior during the initial phase of the opening process is an important technical barrier for researchers in the airdrop community. As a parachute is expected to open successfully at higher speeds and lower altitudes, the ability to predict the stability of the initial interaction between the decelerator and the flow field becomes crucial. A stability analysis of a mathematically rigorous model for this process would predict which conditions would result in various types of stable and unstable behavior. The analysis would provide details about the shape of the canopy just prior to the inflation process. Currently, all models of the opening process (Steeves, 1989; Ross, 1971; Purvis, 1982) apply only after the decelerator has attained some assumed initial shape.

The subject of this paper is the development of a mathematical model of a canopy's behavior just after the canopy is extracted. This model requires the development of a partial differential equation (PDE), which governs the interaction of the canopy with the flow field. The derived model could be analyzed using dynamical system theory to determine the stability of the canopy as a function of the values of various physical parameters (e.g., airspeed and line tension).

There are numerous models in the open literature for the interaction of a flexible structure with a flow field; for an overview see Dowell, 1980. Typically, these models were created to analyze the aerodynamic flutter problem of a structure in a supersonic flow field. Since most Army airdrops are performed at subsonic speeds, these models are not directly applicable. Work has been published for problems where a pipe either conveys fluid or is in a flow field (Paidoussis and Issid, 1974; Paidoussis, 1966). These models were either not applicable, such as those for pipes conveying fluid, or too restrictive, such as the linear models of Paidoussis, 1966. Hence, the development of an applicable model was essential.

The application of dynamical system theory to aerodynamic stability problems is not new. In fact, the supersonic flutter problem has been examined in some detail (Holmes and

Future Directions of Nonlinear Dynamics in Physical and Biological Systems
Edited by P.L. Christiansen *et al.*, Plenum Press, New York, 1993

Marsden, 1983; Dowell, 1966, 1980). The stability of pipes conveying fluid has also been examined by Holmes, 1977. The idea is to use dynamical system techniques to find the "essential generic models" from the full system of PDEs (Holmes, 1977).

PHYSICAL MODEL

The decelerator is modelled as a long thin tube, with mass per unit length m, which is simply supported at each end. The tube is immersed in a fluid which is flowing with a free stream velocity U parallel to the initial (undeformed) axis of the tube. The tube may also be subject to an initial axial load, T_o, applied at the right support by displacing the right support.

The boundary conditions are chosen to model the attachment of the decelerator to the payload at one end and to the extraction chute at the other end. The boundary conditions wherein both ends are fixed or one end is fixed with the other free may also be used.

The material is assumed to be viscoelastic and to obey the Kelvin-Voight model, as assumed by Holmes, 1977 and Paidoussis and Issid, 1974. The axial extension, w(x), induced by the lateral deflection y(x) is given by $\int (y')^2\, dx$.

Paidoussis, 1966 used the result of Lighthill, 1960 for the resultant relative velocity, v(x,t), between the tube and the flow to derive a linearization (first order approximation) to the drag experienced by the tube. To derive a better approximation, it is necessary to examine more closely the drag equation and the geometry of interaction of the tube with the flow field. Taylor, 1952 based his model for the effect of tube inclination to the flow on a curve fitted to laminar flow data in which sine squared worked well. The difficulty with this work is that the sine squared term leads to terms in the PDE which are even in y. This result is not physical. The same data have been fitted well by the current author using sine cubed for the nonlinear extension and this fitting leads to physically meaningful terms for the PDE, i.e., terms which are odd in y.

A new drag equation was derived to include the effect of the tube's velocity in the y direction on both the magnitude and direction of the total fluid velocity relative to the tube and the new nonlinear extension to the drag. Expanding the resulting equation in a Taylor series and retaining the first set of higher order terms, the normal drag force becomes

$$\begin{aligned} F_N &= \tfrac{1}{2}\rho D C_f [U^2 y' + U\dot{y}] \\ &\quad + \tfrac{1}{2}\rho D[U^2(C_{D_p} - \tfrac{1}{2}C_f)(y')^3 + U(3C_{D_p} - \tfrac{1}{2}C_f)(y')^2\dot{y}] \ . \end{aligned} \tag{1}$$

The longitudinal drag force is the one used by Paidoussis, 1966. Thus, the equation of motion may be written as (see Paidoussis, 1966; Holmes, 1977), with simple supports,

$$\begin{aligned} &\alpha\dot{y}'''' + y'''' + [u^2(1 - \tfrac{1}{2}\zeta c_T(\tfrac{1}{2} - x)) - \Gamma - \kappa\int_0^1 (y')^2\, dx - \sigma\int_0^1 y'\dot{y}'\, dx]y'' \\ &\quad + 2\beta^{1/2}u\dot{y}' + \zeta c_T u^2 y' + \tfrac{1}{2}\beta^{1/2}\zeta c_T u\dot{y} \\ &\quad + \ddot{y} + \tfrac{1}{2}\zeta u^2 c_T(R - \tfrac{1}{2})(y')^3 + \tfrac{1}{2}c_T\beta^{1/2}\zeta u(3R - \tfrac{1}{2})(y')^2\dot{y} = 0 \ . \end{aligned} \tag{2}$$

Equation 2 is in dimensionless form. If only the linear terms in equation 2 are retained, the resulting equation is equivalent to the linear PDE examined by Paidoussis, 1966. If the last two nonlinear terms on the left hand side (LHS) of equation 2 are eliminated, the resulting equation is similar to that derived in Holmes, 1977.

DERIVATION OF LOW-DIMENSIONAL MODELS

The eigenfunctions for simple supports are $\sin(j\pi x)$. Using Galerkin averaging (see Dowell, 1966; Holmes, 1977) of equation 2 with $y_n=\sum_{j=1,n} r_j(\tau)\sin(j\pi x)$ gives a set of n second order ordinary differential equations (ODEs) in the time coefficients $r_i(\tau)$. Truncating the series at n = 1, and 2, gives two low-dimensional models:

$\underline{for \quad n = 1:}$

$$\ddot{r}_1 + A_1\dot{r}_1 + B_1 r_1 + \kappa\frac{\pi^4}{2}r_1^3 + \sigma\frac{\pi^4}{2}r_1^2\dot{r}_1 + \frac{\pi^2 c_T u\zeta\beta^{1/2}(3R-\frac{1}{2})}{8}r_1^2\dot{r}_1 = 0 \quad , \tag{3}$$

$\underline{for \quad n = 2:}$

$$\begin{aligned}\ddot{r}_1 &+ A_1\dot{r}_1 + B_1 r_1 + \tfrac{32}{9}\zeta c_T u^2 r_2 - \tfrac{16}{3}\beta^{1/2}u\dot{r}_2 \\ &+ \kappa\pi^4[\tfrac{1}{2}r_1^2 + 2r_2^2]r_1 + \sigma\frac{\pi^4}{2}[r_1\dot{r}_1 + r_2\dot{r}_2]r_1 \\ &+ \pi^2\zeta c_T u^2(R-\tfrac{1}{2})[\tfrac{4}{5}r_1^2 r_2 - \tfrac{144}{35}r_2^3] \\ &\qquad + \pi^2 c_T\zeta\beta^{1/2}u(3R-\tfrac{1}{2})[\dot{r}_1 r_2^2 + \tfrac{1}{8}r_1^2\dot{r}_1] = 0 \quad ,\end{aligned} \tag{4a}$$

$$\begin{aligned}\ddot{r}_2 &+ A_2\dot{r}_2 + B_2 r_2 + \tfrac{8}{9}\zeta c_T u^2 r_1 - \tfrac{16}{3}\beta^{1/2}u\dot{r}_1 \\ &+ \kappa\pi^4[2r_1^2 + 8r_2^2]r_2 + \sigma\frac{\pi^4}{2}[4r_1\dot{r}_1 + 16r_2\dot{r}_2]r_2 \\ &+ \pi^2\zeta c_T u^2(R-\tfrac{1}{2})[\tfrac{176}{35}r_1 r_2^2 + \tfrac{4}{5}r_1^3] \\ &\qquad + \pi^2 c_T\zeta\beta^{1/2}u(3R-\tfrac{1}{2})[\tfrac{1}{2}r_2^2\dot{r}_2 + \tfrac{1}{4}r_1^2\dot{r}_2] = 0 \quad ,\end{aligned} \tag{4b}$$

$$\textit{Where,} \quad A_s = \pi^4 s^4\alpha + \tfrac{1}{2}\beta^{1/2}\zeta c_T u \ , \ B_s = \pi^4 s^4 + [\Gamma - u^2]\pi^2 s^2 \tag{5}$$

RESULTS

This section describes the use of dynamical system theory to investigate the bifurcation behavior of the low dimensional model given by equation 3.

Equation 3 has equilibrium points at

$$\begin{aligned} x_2 &= 0 \ , \\ \frac{\kappa\pi^4}{2}x_1^3 + B_1 x_1 &= 0 \quad . \end{aligned} \tag{6}$$

Equation 6 agrees with the results obtained by Holmes, 1977. If $B_1 > 0$, then (0,0) is a unique fixed point and it is a sink. This means that the undeflected shape is stable. If $B_1 < 0$, there exist three equilibrium points given by (0,0) and $([\pm 2B_1/\kappa\pi^4]^{(1/2)}, 0)$. Unlike the problem examined by Holmes, 1977, a three parameter bifurcation space $(u, \Gamma, R) \in \mathbb{R}^3$ exists because of the new parameter R, defined as $R=C_{Dp}/C_f$. The tube is now unstable since the center fixed point is a saddle and, for R large enough, the other two are sinks (see Holmes, 1977). Note also that the change in sign of B_1 from positive to negative takes place at the Euler buckling mode. For the case where $R < R_{cr}$ (where R_{cr} is the critical value above which the results agree with Holmes, 1977), different results are obtained. For $B_1 > 0$ there still exists a unique fixed point at $(x_1,x_2) = (0,0)$. For $B_1 < 0$ and $R > R_{cr}$, the structure is identical to that found by Holmes, 1977 and there are three equilibrium points, one saddle and two sinks. If, however, R is reduced to a value below R_{cr}, then the two sinks become sources and a pair of limit cycles is born. Physically, these limit cycles correspond to the canopy oscillating about either one of the displaced positions, i.e., fluttering.

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